

GNOMONIQUES,

O R,

The Art of Shadows

I M P R O V E D.

Plainly set forth in the Drawing of

SUN-DIALS

On all sorts of

PLANES

By Different Methods.

With the Geometrical Demonstra-
tions of all the Operations.

By Mr. *DE LA HIRE* of the Royal
Academy of Sciences at *P A R I S*.

*Englified and Illustrated by an Example in
Numbers.*

By *J. LEE K* Mathematician.

The Second Edition.

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MONUMENTS

OF

THE ART OF SURVEYING

IMPROVED

Plainly set forth in the Drawing of

STATION-DIALS

On all sorts of

PLACES

By Different Methods

With the Geometrical Demonstrations

Proofs of all the Operations

By W. D. L. & W. B. of the Royal

Academy of Sciences at Paris

Translated and Illustrated by

W. D. L. & W. B.

By A. & R. K. Mathematicians

The Second Edition

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PREFACE.

I Have always considered the Description of *Sun-Dials* as one of the most ingenious and useful Inventions derived from the Study of the *Mathematics*. Also there is nothing that draws more Admiration from all Men, than to see Strait-lines drawn on a *Plane* at *Unequal Distances*, to measure exactly the equal Divisions of the time of the Continuance of a Day; and altho the *Sun* appears in different places of Heaven according to the different Seasons of the Year, yet the same Strait-lines do still determine the same hour at all these different Seasons.

But these Hour-lines have nothing in them that deserves to be considered, if we compare them to those which trace forth the way which the *Sun* makes in his Distances from the Equinoctial Line, which questionless have given place to the more profound Meditations.

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tions of the *Sections* of a *Cone* ; which at this day is the most considerable part of our *Speculative Geometry*.

I might easily demonstrate that we are beholding to *Sun-Dials* for the discovery of those admirable *Curve Lines* whereof we find very great use in all parts of the *Mathematicks* ; for we cannot consider the Shadow of the end of any body pointed on a *Plane*, without perceiving at the same time the *Curvature* which the way of the *Sun* traces forth ; which is most like to that of the *Section* of an upright *Cone* which hath a *Circle* parallel to the *Equinoctial* for its *Base*, on which we may suppose the *sun* moves then when he makes that Shadow.

But altho the *Properties* of these *Curve Lines* serves as a *Foundation* for the most part of the *Descriptions* of *Dials*, yet in this *Work* I do not intend to explain any thing thereof in particular, for that would be to depart too much from my *Subject*.

Seeing also that divers other knowing *Geometers* have largely treated there-

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thereof, it seems to me that it will be very useless to re-search into the time of the most ancient Antiquity, and who was the Inventer of this Art : We may only believe very likely that it was perfected by little and little, and that the first Men seeing the necessity which they had to divide the continuance of a day into divers parts, judged that it could not be better done than by the same *Sun* which limits the continuance thereof.

We may also be very easily perswaded that the Meridian Line hath been the first which they have drawn, as well because that it divides the apparent day, which is the time during which we may see the *Sun*, into two equal parts, and that the Shadow of a Strait-line raised perpendicularly upon an Horizontal Plane, was always extended along this Meridian Line when the top of it meets with it, although it be in different places ; so that it serves us to know the greatest height of the *Sun* above the Horizon every day, the which changes during the time of half a Year.

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It seemed to them one of the most considerable Phenomena's of the *Sun*.

The most part of the Ancients divided the Space of time (which is from the rising to the going down of the *Sun*) into twelve equal parts , which they called Hours , and they began their Account from *Sun* rising ; and although the Hours changed their length during the half year to those which did not inhabit under the Equinoctial Line , where the apparent days are always equal to one another , therefore they have always Mid-day at the Sixth Hour : but these sorts of Hours have no Circles of the Sphere that represent them.

The *Babylonians* began the day at the *Sun* Rising , and divided the Duration into 24 equal Hours : The *Italians* began it at *Sun* Setting , and made also their Hours equal : These two manners of beginning the day have the Horizon for their term ; the first did always know how long time they had from the *Sun* Rising above the Horizon , and the others did always know that which remains to the *Sun* Setting.

The

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The Astronomers and the greatest part of the Nations of *Europe* begin their Day when the Sun comes to the Meridian, these last when the Sun comes to the Meridian under the Horizon; and the first when the Sun comes to the Meridian above the Horizon; this manner of beginning the day has great advantages beyond other.

I undertake in this Work only the Description of the Astronomick Hours, which have for their Term the Meridian, and of the *Italians* and *Babylonians* that begin from the Horizon. In the Construction of these sorts of *Dials* there are two principal Operations which we may consider each in particular, which has obliged me to divide this Treatise into Two Parts.

In the First Part, after I have explained as brief as I can, that which is necessary to be known for the right understanding of the Construction of *Sun Dials*, in which I give the definition with that of all the parts: I propose following divers

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Manners and Practices, to draw their principal Line, with the Points which are necessary for the description of the Hour Lines, to the intent that we may serve our selves with those which are most fit in the different Rencontres of the *Planes* proposed.

Each of these Practices have advantages in particular Cases; I observe by which we may serve our selves very near following the Exposition of the *Plane* proposed, to the intent that those that have not sufficient knowledge of the different Rencontres of *Planes* with the Circles of the Sphere, may not give themselves the trouble to follow a Method from which they cannot draw a great advantage.

In all these Practices which I propose, I make no use of the Magnetical Needle, for there happens great change to the Variation of the Magnetical Needle; besides we are not assured that there is no Iron hid, or some Stone or Brick which is of the Nature of Iron, which may turn aside the Needle

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Needle from its true Direction towards the place whither it would go if it were free.

Also I approve not that Method which many do practice to find the Declination of a *Plane*, that is to say, the Angle which the Meridian Line makes on a Horizontal Plane with the Horizontal Line, which is the meeting of the Plane of the *Dial* with that Plane: they draw on a Plane set Parallel to the Horizon (the which we call a *Level Plane*) a Meridian Line following one of the Practices, which may be seen hereafter; and when the Sun marks Mid-day on that Horizontal Plane, they mark a Point of Shadow on the proposed Plane; but you must observe that the Errors that one commits in all the Operations, as well in the placing of the Plane level, as in the determination of the Meridian, are multiplied and increased in transporting them to another *Plane*.

For the same Reasons we ought also to reject all sorts of Instruments, unless they be very plain and very large:
There.

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Therefore I have thought good to use only the Ruler and Compass, the Plomb Line and the Level, and to draw the Lines and Circles only upon the given *Plane*.

Altho we may determine the length of the Lines, and also the most part of the Angles by Calculation of Spherical and Strait-lin'd Triangles, which serves for the drawing of *Dials*, Yet I have thought good that I ought not to propose any (of those ways) in this small Treatise, because that the most part of those Calculations are much longer than the Practice, and they are founded but upon the same Angles, and the same Lines which I have used in the Practice; so that those that can use Calculation, shall find no great difficulty to apply them to Numbers and Sines, where I only propose Lines and Angles.

After I have taught to Draw the Principal Line of *Dials*, I proceed to the Second Part, wherein I shew to draw

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draw the Astronomical Hour Lines, and then to describe the Parallels of the Signs.

I propound no particular Construction on the Horizontal and Vertical Planes, which only gives particular Rules for each Case, the which in the ordinary way happens very seldom: Therefore the Methods that I teach are for all sorts of Planes indifferently considered. I know well that there are divers Cases where we might find Abridgments, but these Abridgments consists only in certain Lines and Points which come to be united in the general Practices which I give: Also it is to be observed that the Portions of Curve Lines that I describe, are always Conick Sections, that is to say, either Ellipses, Hyperboles, or Paraboles, and sometimes Circles, when the Plane is perpendicular to the Axis of the Cone, the which is always an upright Cone.

The most part of the Practice that I teach being founded on the Declination of the Sun, I give you a Table

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ble thereof Calculated for the days
of Four Years following one ano-
ther, to the end to comprehend the
Bisextile or Leap Year; I there al-
so add the Differences of the De-
clinations for every day, with a Table
of Latitudes of the Principal Cities
of the World.

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G N O.

GNOMONIQVES,

OR

The ART of DRAWING
SUN-DIALS

On all Sorts of

PLANES.

THE FIRST PART.

C H A P. I.

*Of the Circles of the Sphere necessary
to be known for the Drawing of*
SUN-DIALS.

THE *Sphere* is an Instrument whereby we explain the *Daily Motion* of the *Celestial Bodies*, according as they appear to us to move always from *East* to *West*, and also the proper *Motion* of the *Sun*, which moves
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from *West* toward the *East*, and makes his *Revolution* thro the *Twelve Cælestial Signs* in the space of one year.

This *Instrument* is composed of divers *Circles*, of which we only describe those that belong to our present *Subject* : Those *Circles* whose *Planes* pass thro the Center of the *Earth*, are called *Great Circles* of the *Sphere*, and all the other are *Less*.

But before we speak of these *Circles*, we ought to consider the *Axis* of the *Sphere*, which we conceive to be as a strait line about which the *Instrument* is turned. The two Ends of the *Axis* are called *Poles*, the one *North* and the other *South*.

The *Earth* is placed in the middle of this *Instrument*, and consequently the *Axis* passes thro the Center thereof. We may understand from *Astronomical Observations* that the *Globe* of the *Earth* is so little, in respect to its distance from the *Sun*, so that we may consider it as a *Point*, if we compare it to that distance.

The *Equinoctial* or *Equator* is a *Great Circle*, and one of the chiefest of the *Sphere*, the *Plane* whereof is at *Right-angles* to the *Axis*, it divides the *Sphere* into two equal parts; whereof one is called *Septentrional*, and the other *Meridional*.

Fig. 1. The *Ecliptick* is another *Great Circle* whose *Plane* makes an angle with the *Plane* of the *Equinoctial* of 23 degrees 30 minutes; the *Sun* moves under this *Circle*, going from the *West* toward the *East*, and makes one intire *Revolution* in

in 365 days and near 6 hours. The Inclination of this Circle to the *Equinoctial*, causes the different *Declinations* of the *Sun* in regard to the *Equinoctial*; it is divided into Twelve equal parts, which are called *Signs*: And we begin from the Intersection thereof with the *Equinoctial*, proceeding towards the *North*. The Names of the Twelve *Signs* are *Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, *Pisces*. Their Characters are γ , δ , Π , $\ominus$, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, $\mathfrak{F}$, $\mathfrak{G}$, $\mathfrak{H}$.

The *Tropicks* are two Circles parallel to the *Equinoctial*, which touches the *Ecliptick* in the point of its greatest distance from the *Equinoctial*; therefore these Circles are distant from the *Equinoctial* 23 deg. 30 min. on one side toward the *North*, and on the other side toward the *South*.

So that it is manifest, That when the *Sun* is in the common Intersection of the *Ecliptick* and *Equator*, the Motion of the *Sphere* about its *Axis*, which goes from *East* to *West*, and is called the Motion of the *Primum Mobile*, makes him appear to us in the *Equinoctial*; and also when he is in his greatest distance from the *Equinoctial*, the same Motion of the *Primum Mobile* makes him to appear to us to move in the *Tropicks*.

The *Zenith* is an imaginary Point in the *Sphere*, marked by a strait Line coming from the Center of the *Earth*, and passing by some place of the Superficies thereof. This Line is called the *Vertical Line* of that place.

The *Horizon* is a Great Circle, whose Plane cuts the *Vertical Line* at Right-angles. The *Horizon* of a Place distinguishes the visible part of Heaven of that Place, from that part of Heaven which is not there seen.

The *Meridian* is a Great Circle which passes thro the *Poles* and *Zenith*, the Plane whereof is at Right-angles with the Planes of the *Equinoctial* and *Horizon*. When the *Sun* comes to this Circle, he is in the middle of his apparent Course during a day, and is at his greatest height above the *Horizon*, because this Circle passes thro the *Zenith* and *Poles*.

If we suppose the *Equinoctial* to be divided into 24 equal parts, beginning from the *Meridian*, the 6th and 18th part shall fall on the Intersections of the *Horizon* and *Equinoctial*, because the *Meridian* and *Horizon* are at Right-angles to one another; and if we imagine other Circles like the *Meridian*, that is to say, that pass thro the *Poles* of the *World* and Point of Division of the *Equinoctial*, those Circles which we call *Meridians*, shall be the *Hour Circles*, among which is the *Meridian* of the Place, whereof all the Planes intersect one another in the *Axis*. We may also conceive others, which divide each part into two or four, to mark the half-hours and quarter-hours; for if we suppose these Circles to be fix'd, then when the *Primum Mobile* turns the *Sun* with his *Ecliptick* about the *Axis*, the time of his apparent Course shall be divided into hours, halves, and quarters, by these *Meridians*. Also

Also we number the *Declination* of the *Sun* upon the like *Meridians*, which do all intersect the *Equinoctial* at *Right Angles*, which we make to pass thro the *Center* of the *Sun* in the *Ecliptick*: We number this *Declination* from the *Equinoctial* towards the *Poles*: Therefore it is either *South* or *North*. The *Angles* of *Declination* are measured by *Arches* of *Circles*.

Those *Circles* that pass by the *Vertical Line* are called *Vertical Circles* (or *Azimuth*) and their *Planes* are perpendicular to the *Plane* of the *Horizon*; they serve to measure the height of the *Sun* above the *Horizon*, which is numbred from the *Horizon* toward the *Zenith*.

It is manifest from that which has been said before, that there are infinite *Horizons* and *Meridians*, and that there are only these two great *Circles*, which may change according to the different places on the *Earth*, for they are established by the *Vertical Line*.

The *Amplitude* of *Rising* or *Setting* is counted on the *Horizon*, beginning from the *Points* where the *Equinoctial* cuts the *Horizon*, and is numbred toward the *South* or *North*.

If we conceive that in the *Revolution* of one day the *Horizon* moves, as being fastened to the *Axis*, so as it cannot change its *Inclination*, then when it shall pass by the 24 equal *Divisions* of the *Equinoctial*, it shall represent the 24 *Circles* of the *Italian* or *Babylonian Hours*.

CHAP. II.

*Of the Definition of Sun-Dials , and
of the principal parts which serves
for their Construction.*

THe Distance from the Center of the Earth to the Superficies thereof not being considerable, in respect of the Distance of the Earth from the Sun, we may take any Point on the Superficies and consider it as its Center in relation to the Motion of the Sun.

Therefore if we place a Style which is a Pointed Rod upon any Plain Surface , and then consider the Point of that Style as the Center of the Earth, the Intersections of that Surface with the Planes of the Hour Circles , of the Equinoctial or Equator , of the Horizon and of the other great Circles, shall be strait Lines, which retain the Names of the Planes of the Circles from whence they were produced : All these Lines on that Plane Surface with the Style makes a Sun-Dial.

The Shadow of the Point of the Style , which is one of the Points of the Axis shews the Hours.

And if the Axis which passes by the Point of that Style meets with the Plane of the Dial in any Point , that Point is called the Center of the Dial;

Dial; for it is evident that all the *Hour Lines* shall meet in that *Point*.

It is also evident that the Shadow of the top of the *Style* gives the *Hours*, and shews when the *Sun* meets with any one of the *Circles* of the *Sphere*; for when the *Sun* comes to a great *Circle*, the Shadow of the *Axis* is extended in the *Plane* of that *Circle*, if that *Circle* passes by the *Axis*; and if it passes not by the *Axis*, the shadow of the *Point* of the *Style* shall be in the *Plane* of that *Circle*; for the *Planes* of great *Circles* pass by the *Point* of the *Style*.

And if we conceive a *Conical Superficies* which has for its *Base* a less *Circle* of the *Sphere*, and for its *Vertex* the *Point* of the *Style*, that *Conical Superficies* shall meet the *Surface* of the *Dial* in a *Curve Line*; so as when the *Center* of the *Sun* shall touch that less *Circle* which is the *Base* of the *Conical Superficies*, the Shadow of the *Point* of the *Style* shall touch the *Curve Line* which is the meeting of that *Curve Superficies* with the *Plane* of the *Dial*: for the *Point* of the *Style* is on that *Superficies* whereof it is the *Vertex*.

The *Foot* of the *Style* is that *Point* on the *Plane* of the *Dial*, which is the meeting of a strait *Line* drawn perpendicularly to that *Plane*, and which passes by the *Point* of the *Style*.

If the *Plane* of the *Dial* be considered as the *Plane* of the *Horizon* of any place, the strait *Line* that passes by the *Point* of the *Style* and by its *Foot*, shall be the *Vertical Line* of that *Place*:

and the *Plane* that passes by that *Vertical* and by the *Axis*, shall be the proper *Meridian* of that *Place*, considered as the *Horizontal* of a *Place*.

The meeting of the *Meridian* and *Surface* of the *Dial* is called the *Substyler Line*, or the *Meridian* of the *Plane* or *Surface* of the *Dial*, which we ought to distinguish from the *Meridian* of the *Place*, which is the meeting of the *Meridian* proper to that *Place* and of the *Surface* of the *Dial*, at least if they be not coincident, which happens when the *Dial* does not decline from the East or West.

We see by the position of these *Lines* that the *Substyler Line* is always at *Right Angles* with the *Equinoctial Line*.

We ought to make the *Dial* so as the Foot of the *Style* be not incumbered, for that *Point* serves for many *Operations*; therefore the *Style* must be planted a little obliquely upon the *Surface*.

I understand by the length or height of the *Style* the strait *Line* drawn from the *Point* to the foot thereof.

The *Arches* of the *Signs* on the *Surface* of the *Dial*, are the *Descriptions* of the *Parallels* to the *Equinoctial*, which pass thro the 12 equal *Divisions* of the *Ecliptick Line*, which shew the beginning of the *Signs*.

CHAP. III.

To Mark the Points of Shadow.

Fig. 2. **T**He Practices which are taught in this Treatise, being founded on the *Points of Shadow* of the *Point* of the *Style* to be very small, it is very necessary to mark them very exactly; but it is very difficult because of the *Penumbra*: See here two ways by which it may be done.

The first is to fit a small round *Plate* to the *Point* of the *Style*, which may be parallel to the *Plane* of the *Dial*, whereof the *Center* may be joyned to the *Point* of the *Style*; then having drawn the *Shadow* of the said *Plate* on the *Plane* of the *Dial*, take the middle of that *Shadow*, which shall be the *Shadow* of the end of the *Style* at the same time when we observed the *Shadow* of the *Plate*.

The Second way is to make a small round hole in a little piece of *Past-board* or thin *Plate*, or other like body, and having applied it to the end of the *Style*, so as the *Center* of the *Hole* may be joyned to the *Point* of the *Style*, and that the small *Plate* may regard the *Sun* perpendicularly; the light of the *Sun* shining thro the
Hole

Hole shall mark a clear Circle or Oval D E in the Shadow of the Plate on the Plane of the *Dial*; which we draw on the said Plane; and if it be an Oval, having drawn a strait Line D P E from the Point P, which is the foot of the *Style*, whereof S is the Point, which may pass thro the *Center* of that Oval and cut it in D and E, or draw D G and E F parallel to one another, and making any *Angle* with D E, D G being made equal to D S, and E F equal to E S, the Line G F shall cut D E in the Point A, which shall be the Shadow of the Point of the *Style* S, at that time when the Oval was drawn: But we may take the *Center* of the Oval for the Point A, without falling into any sensible Error, as we may see by this Operation.

But if the Shadow be a Circle, the *Center* of that Circle shall be the Shadow of the Point of the *Style*.

Here Note, That we ought to make the Hole as small as is possible, for so the Operation shall be more exact. It is sufficient that we may see distinctly the *Figure* of the Light in the Shadow of the small Plate.

CHAP. IV.

To Draw the Horizontal Line.

A *Style* the Point whereof is S, being planted upon the *Plane* of the *Dial*, we apply a Rule AS, so as one of the Edges thereof AS may be level, touching the Point of the *Style*, and that the end of that Edge of the Ruler may touch the *Plane* of the *Dial* at the Point A, which shall be one of the Points of the *Horizontal Line*.

By the Point A draw a level Line on the *Plane* of the *Dial*, which shall be the *Horizontal Line*.

There are too many ways to set a Ruler level, that I will prescribe none in particular.

CHAP.

CHAP. V.

To find the Substylar Line, two Points of Shadow being given in a certain condition.

To find the Center and draw the Equinoctial Line, the Declination of the Sun, and one Tract of Shadow being given.

Fig. 4. **L** Et there be a *Style*, the Point whereof is S, and P the Foot, having marked the Point A, which let be the Shadow of the Point S, on the *Plane* of the *Dial*; on the Point P as a *Center*, and at the Distance P A, describe the Circle A B, and when the Shadow of the Point S comes again to the Circle A B on the same day, at the Point B we mark that Point. Then draw the Line A B, and divide it into two equal parts in D, the strait Line P D, shall be the *Substylar Line*, which ought to be at *Right Angles* to the Line A B. We may take divers Points, as A, for to find divers Points as B, and if the Operation be made exact, all the Points as D, ought to be in one and the same strait Line with the Point P.

If the *Suns Declination* has changed considerably in the time which has passed between the two Observations of the Points of Shadow A and B; which may fall out when the *Sun* is near the *Equinoctial* Points, or when the *Style* P S is B;

very high; or lastly, when there has passed much time between the Observations, we shall not have the *Substylar Line* exactly.

The Demonstration of the former Practice.

Supposing the Declination of the *Sun* not to be changed between the two Observations of the Points of Shadow A and B; by construction the two *Triangles* A P S, B P S, are equal and alike; therefore the *Angle* P S A is equal to the *Angle* P S B; the *Sun* therefore was equally elevated above the *Plane* of the *Dial*, then when the two Points of Shadow were observed. Therefore the *Sun* was equally distant from the *Meridian* of that *Plane* at the times of the two Observations: Therefore the Line P D, which divides the Line or Arch A B into two equal parts, shall be the *Meridian* of the *Plane*.

To find the Center of the *Dial*, and to draw the Equinoctial Line, knowing the place where the Shadow of the Point of the *Style* cuts the *Substylar Line*.

If between the Points of Shadow A and B, we mark a Succession of Points of Shadow, so that we may have the Point L where the Shadow of the Point of the *Style* meets with the *Substylar Line* between the Observations, having erected P F perpendicular to the *Substylar Line*, and equal to the height of the *Style* P S, let the straight Line F L be drawn, and having made the *Angle* L F E equal to the Declination of the *Sun* at that

that time when the Point of Shadow *L* did mee with the *Substylar Line*, so as the Point *E*, or the meeting of the *strait Line* *FE* with the *Substylar Line*, may be always towards the Convex part of the *Curvature* of the Line *ALB*, that Point *E* shall be the Point where the *Equinoctial Line* *EG* cuts the *Substylar Line*; which Lines shall intersect one another at *Right Angles*.

Then having drawn *FC* perpendicular to *FE*, the *strait Line* *FC* determines the position of the *Axis* with the *Substylar Line*; and if it meets with it in the Point *C*, that Point shall be the Center of the *Dial*, and the Line *FC* shall be the *Angle* of *Inclination* of the *Axis* with the *Substylar Line*, which serves to place the *Axis*, and to find the other necessary Points for the Construction of the *Dial*.

It is not necessary that the Point of Shadow *L* should be taken on the same day when we observed the other Points *A* and *B*, it is sufficient that we have the Declination of the *Sun* then when we make Observation of the Point *L*, and on which side the convexity of the *Tract* of the Shadow shall be on that day, to find the Point *E*.

Demonstration.

The *Demonstration* of this Operation is manifest, if we consider that we have made the *Angle* *LFE*, and that the Line *FC* ought to be extended in the *Plane* of the *Meridian*, which is perpendicular

lar to the *Plane* of the *Dial*, and that the meeting of the Line *DPC* and the *Point F*, ought to be conjoynd in the *Point* of the *Style S*.

Another way of finding the Substylar Line by the Amplitude of the Suns Rising and Setting upon the Plane of the Dial.

When the *Sun* begins to rise on the *Plane* of the *Dial*, mark the Shadow of a Small Thread extended from the Foot of the *Style* to its *Point*, and do also the same when the *Sun* sets on the *Plane* of the *Dial*, the *Angle* comprehended between these two Lines of Shadow, whose Vertex is at the Foot of the *Style*, being divided into two equal parts, shall give the *Substylar Line*.

This is manifest, for that *Angle* is the Sum of the Ortive and Occasive Amplitudes of the same day, which we suppose to be equal.

The *Substylar Line* being placed, we may find the Center of the *Dial* by the Practice of the 11th Chapter, using only one Point of Shadow, and knowing the Declination of the *Sun* at the Hour where the Point of Shadow has been marked; and if the *Dial* have no Center, we may have the Inclination of the *Axis* with the *Substylar Line*, which shall serve instead of the Center for the placing of Hours: Also by the same Practice we may have the position of the *Equinoctial Line*.

CHAP. VI.

To place the Substylar and Equinoctial Lines, and the Center of the Dial, and to determine the position of the Axis.

Any Two Points of Shadow being given, with the Declination of the Sun at the time of Observation of the Points of Shadow.

Fig. 5. **A** Style being placed on the Plane of the Dial, whereof the Point may be S and P the Foot, and any two Points of Shadow A and B, taken at pleasure.

Upon some certain Plane having made the *Angle dfa* equal to the Sum or Difference of a *Right Angle*, and that of the Declination of the Sun on that day on the which the Points of Shadow were mark'd, according as the Declination is North or South; for you would have a Point of the *Substylar Line* as Q which may answer to a Point of the *Axis*, which may be more North than the Point of the *Style*, you must make the *Angle dfa* equal to the Sum of a *Right Angle* and *Angle* of the Declination of the Sun, if the Declination be North, but equal to the Difference of a *Right Angle* and *Angle* of the Declination if it be South.

Having

Take sd of any length, and make sa and sb equal to the *Intervals* SA , SB , from the *Point* of the *Style* to the *Points* of the *Shadow*; from the *Point* A as a *Center* at the distance ad , describe Two *Arches* of *Circles* at T and L , and from the *Point* B , at the distance of the *Line* bd , intersect the former *Arches* at the *Points* L and T , then draw the *strait* *Lines* LT and AB , which shall intersect one another at *Right Angles* in the *Point* O ; and from the *Point* O as a *Center*, at the distance LO or OT , which are equal, describe the *Semicircle* LDT .

Then draw the *strait* *Line* PGK parallel to AB , intersecting LT in the *Point* G , and make GK equal to PS the height of the *Style*: and from the *Point* P , as a *Center*, at the distance of the *Line* sd , describe the *Arch* IE , cutting the *strait* *Line* LT in the *Point* I .

And from the *Point* K as a *Center*, and at the distance GI , describe the *Arch* DR , cutting the *Semicircle* LT in the *Point* D : Then to the *Line* LT , let fall the perpendicular DQ , and draw the *strait* *Line* QP , which shall be the *Substylar* *Line*.

If the *Point* Q falls too near to the *Point* P , we may take sd greater, and begin the work again to determine the *Substylar* *Line* more exactly.

If the *Declination* of the *Sun* be considerably changed between the *Observations* of the *Points* of *Shadow*, be it that the *Observations* be made on the same day or on different days, the *Angles*
 $d\angle a,$

$\angle f a$, $\angle f b$ must be made according to the different *Declinations* at the times of *Observation*, and the *Angle* $\angle f a$ being made as has been taught before for the time of the *Observation* of the *Point* of *Shadow* A, make $f a$ equal to S A; also the *Angle* $\angle f b$ being made for the *Observation* of the *Point* of *Shadow* B, make $f b$ equal to S B.

Then from the *Points* P and Q of the *Substylar Line* raise the *Perpendiculars* P N equal to the height of the *Style* P S, and Q M equal to Q D; then if the *strait Line* M N being drawn meet with the *Substylar Line* in C, the *Point* C shall be the *Center* of the *Dial*, and the *Line* N E perpendicular to N M, meeting with the *Substylar Line*; the *Line* V E being drawn perpendicular to the *Substylar Line* passing thro the *Point* E shall be the *Equinoctial Line* of the *Dial*.

We may observe that the *Point* D ought to be on the side with the *Semicircle* T L G, which is cut by the *Line* P G K, and which answers to the *Point* M of the *Axis*, which is supposed to be more toward the *North* than the *Point* S; the *Point* D may indifferently meet with the *Circle* on either side of the *strait Line* T L.

Also, We may observe that if the *strait Line* L T passes by the *Point* P, that *Line* shall be the *Substylar Line*, and it shall always pass by the *Point* P, if the *Points* of *Shadow* A and B are *Points* of the *Equinoctial Line*.

DE.

Demonstration.

It is clear by this Construction that the Line sd represents the Axis, and sa and sb the Lines of the Shadow from the Point of the Style, which make with the Axis sd an Angle equal to the Sum or Difference of a Right Angle and Angle of the Declination of the Sun, according to that which hath been prescribed in the Practice: Therefore if we conceive that the two Triangles asd , bfd , each apart have their Points a and b in the Points of Shadow A and B , and their Points s joyned together with the Point S of the Style, if the Points d of each of these Triangles are also joyned together, these two Triangles in this Position compose a Pyramid $ABSd$ whereof the Line Sd is the Axis of the Dial.

But to find on the Plane of the Dial a Point which may answer perpendicularly to the Point d , as P answers to the Point S , we must consider in the Pyramide the Triangle ABD to move upon the Line AB , so as it is manifest that the Point d describes a Circle such as is TLD , on a Plane which is perpendicular to the Plane of the Dial; and the Intersection of those two Planes is the strait Line TL , which ought to cut the Line AB , which conjoyns the Points of Shadow A and B in the Point O , and OT , OL shall be equal: There remains nothing more but to mark the Point d or D upon the Circle LDT couch'd on the Plane of the Dial.

The Plane of the Circle LDT being perpendicular to the Plane of the Dial, if we conceive that the strait Line sd moves upon its extreme, s being immovable, and put upon the Point of the Style S , and that the other extrem thereof d , may be always in its moving upon the Plane of the Circle LDT , it is evident that that extrem d shall describe a Circle RD upon that Plane which shall have for its Center the Point K , where the Line SK drawn from the Point of the Style S perpendicularly to the Plane LDT , meets with the same Plane, the which Line SK shall be equal to PG : but the Semidiameter of the Circle RD shall be equal to GI if PI be made equal to sd ; but the intersection of the Circle LDT with the Circle RD , which is the Point D , determines the position of the Point d of the Axis. So as DQ considered as perpendicular to the Plane of the Dial, coming from the Point d or D , does there mark the Point Q , which is one of the Points of the Substyler Line, seeing that the Point d is one of the Points of the Axis.

It is also evident that the Plane $NMCP$ being supposed to be perpendicular to the Plane of the Dial, the Line NM does there represent the Axis in its position, the Point C the Center of the Dial, and consequently the Line VEV shall be the Equinoctial.

Ano-

*Another Practice upon the same Positions and
Constructions.*

The same Preparations being made as before, take two small Rods of any firm Matter, as of Wood of a sufficient thickness, or of Iron: and make them pointed at the ends, and equal in length to the strait Lines ad , bd ; it is not material whether they be strait or crooked, if the Distances between their Points be equal to ad and bd .

Put one of the *Points* of that *Rod* which is equal to ad on the *Point* of *Shadow A*, and one of the *Points* of the other *Rod* to the *Point* of *Shadow B*, and joyn them together by their other *Points*, but so as the *Points* that are joyned together may approach or fall back from the *Point* of the *Style* without altering the other *Points* of the *Rods*, which are set on the *Points* of *Shadow A* and *B*; then we take with the *Compasses* or otherwise, the distance between the *Points* a and d , and set that distance between the *Point* of the *Style* and the *Points* of the *Rods* that are joyned together, so as that *Point*, it may be more toward the *North* than the *Point* of the *Style*: by this means the common *Point* of the *Rods* being fixt, shall be one of the *Points* of the *Axis* which ought to pass by the *Point* of the *Style*, therefore the situation of the *Axis* shall be determined.

By this *common Point* of the *Rods* so fixt, which I call *D*, having drawn a *Line* perpendicular to the *Plane of the Dial*, which shall meet it in the *Point Q*, the *Line P Q* shall be the *Substylar Line*.

The *Point C* on the *Plane* of the *Dial* where it is met by the *Line D S*, drawn by the *Point* of the *Style S*, and by the end of the *Rod D*, shall be the *Center* of the *Dial*, and by the *Practice* of the *11th Chapter* we may draw the *Equinoctial Line*: but if we have not the *Center*, we may draw it by the *Practice* of the same *Chapter*.

This Practice may serve to make you understand more easily the Demonstration of the Construction which is proposed in that Chapter.

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CHAP. VII.

To place the Substyler Line, the Center of the Dial and the Equinoctial Line.

One only Point of Shadow being given, with the Declination of the Sun and height of the Pole above the Horizon.

Fig. 6. **H**AVING placed a Style upon the Plane of the Dial, whose Point may be S, and P the foot, and A one Point of Shadow, draw a Horizontal Line by the Practice of the 4th Chapter. And by the Point P draw the Lines BPH perpendicular to the Horizontal Line *h* H, and PZ parallel to *Hh* and equal to the Height of the Style PS. Then from the Point H, where PH meets with the Horizontal Line, draw HZ and ZB perpendicular to ZH, which shall meet with HP at the Point B if the Horizontal Line passes not thro the Point P: First let it meet at the Point B.

Upon some Plane make the Angle *d* *f* *a* equal to the Sum or Difference of a Right Angle, and

of the Declination of the *Sun* at the time when the Point of Shadow was observed, according to the Precepts which have been given in the *6th Chapter*, and make the *Angle d f b* equal to the Sum of a *Right Angle* and the height of the *Pole* above the *Horizon*.

Then taking the *Point d* at pleasure on the *Line f d*, make *f b* equal to *Z B* and *f a* equal to the length of the Shadow from the *Point* of the *Style S* to the *Point of Shadow A*, and draw the *strait Lines a d, b d*.

And by the *Points A* and *B* draw the *strait Line A B*, and from the *Point B* as a *Center*, at the distance *b d* describe the *Arch f L* either above or below the *Line A B*; and likewise from the *Point A* as a *Center*, and at the distance *a d*, describe the *Arch g d*, cutting the *Arch f L* at the *Point L*, and from the *Point L* draw the *strait Line O L* perpendicular to *A B*.

From the *Point O* as a *Center*, at the distance *O L* describe the *Arch D L*.

And from the *Point P* draw the *strait Line P G K* perpendicular to *O L*; and from the same *Point P*, at the distance *d f*, describe the *Arch I* either on the one or the other side of *G*, cutting the *Line L O* at the *Point I*.

Then make *G K* equal to *P S* the height of the *Style*, and from the *Point K*, at the distance *G I* describe the *Arch R D*, cutting the *Arch D L* in *D*, and from the *Point D* draw the *strait Line D Q* perpendicular to *L O*, and the *Line P Q* which

which passes thro the Points P and Q shall be the *Substylar Line*.

If the Point Q be too near to P, we may find another, by taking another Point *d* on the *Line* *d*, as has been said in the Practice of the Precedent Chapter; also we must have a regard to the other *Observations* which have been made upon the same Practice, by that which is there founded on the same Principles.

Consequently we place the *Equinoctial Line* and the *Center* of the *Dial* by the Practice of the 11th Chapter: but we have also here this advantage, that the *Line* which passes thro the Point B and thro the *Center* of the *Dial*, shall be the *Meridian Line*.

In the second place, if the *Horizontal Line* passes thro the Point P, or if the Point B be too far distant from the Point P, we must fasten another *Style* upon the *Plane* of the *Dial*, whereof the Point may pass by the *Line* of the *Plummet* hangd from the Point S of the *Style*, the Point of that second *Style* being called B, we perform the *Operation* as before to find the *Lines* *da*, *db*: but we may use the small *Rods*, as has been taught in the fore-going Chapter, otherwise the *Operation* would be too long.

Demonstration.

This Operation is so like to the precedent; that the Demonstration does not much differ from it; for here instead of a Second Point of Shadow we have

have the Point B , and the Point bs of the Triangle bfd being applied in BS , that Triangle moving upon BS and meeting the other Triangle afd whereof the Points af are applied in AS , and which moves upon AS , so as the Points dd of the two Triangles may be joyned together; then the Axis which ought to pass by the Points Sd , shall be stayed in that Position, as we have seen in the precedent Chapter: for in all the different Positions of the Triangle bfd moving upon BS , the Line Sd which represents the Axis, remains always elevated above the Horizon, with an Angle equal to the Elevation of the Pole above the Horizon, and it shall not be stayed in that Position, but by the meeting of the other Triangle afd , so as the two Points dd of the two Triangles may be joyned together.

The rest of this Operation being altogether like to the precedent, we shall not here again repeat the Demonstration.

CHAP. VIII.

*To find the Center of the Dial , and to
situate the Substylar and Equinoctial
Lines.*

*One only Point of Shadow being given,
and the shortest Shadow.*

Fig. 7. **H**AVING placed a *Style* on the *Plane* of the *Dial*, whose *Point* let be *S* and *Foot* *P*, and marked a *Point* of *Shadow* *A*, and divers other *Shadows* following one another, as *GB*, which may determine the length of the shortest *Shadow*, which is *PB* the *Semidiameter* of a *Circle*, that has for its *Center* the *Foot* of the *Style* *P*, which touches the *Tract* of the *Points* of *Shadow* *GB* in the *Point* *B*, and determines it exactly; which cannot be done by the meeting of the *Two Curves*, that is of the *Circle* *NB* and *Tract* of *Shadow* *GB*:

Therefore upon some *Plane* *ps* equal to the height of the *Style* *PS*, draw *cpb* perpendicular to *sp* and make *pb* equal to *PB*, and *sb* shall be equal to *SB*; and prolong *sb* to *a*, and let *sa* be equal to the length of the *Shadow* *SA*.

Then

Then make the *Angle asc* equal to the Sum or Difference of a *Right Angle*, and the *Angle* of the *Declination* of the *Sun* at the time when the *Points* of Shadow were observed, following that which has been noted in the *Practice* of the 6th *Chapter*, the Line *sc* meeting *bp* in *c* draw the strait Line *ac*.

Then on the *Dial* from the *Point P*, at the Distance *pc* describe the *Arch CF*, and from the *Point* of Shadow *A*, at the Distance *ac* describe the *Arch DC*, intersecting the *Arch CF* in the *Point C*, the *Point C* shall be the *Center* of the *Dial*, and the *strait Line CP* shall be the *Substylar*, which shall cut the *Tract* of Shadow in the *Point B*, where the *Circle NB* ought to touch it.

The *Substylar Line* being placed, we may draw the *Equinoctial Line* by the *Practice* of the 11th *Chapter*.

Demonstration.

This *Practice* is founded on the same Principles as the fore-going Operations, except that in this the Line *sc* is the length of the *Axis* from the *Point* of the *Style S* to the *Plane* of the *Dial*, for the *Triangle cfb* moving upon the Line *sp* applied to *SP*, rests in that Position as it ought to do to determine the *Center C* by the *Triangle asc*, which moves upon *a* applied to *AS*: but instead of these two *Triangles* it sufficeth that the Lines *pc* and *ac* moving

moving on the Plane of the Dial about the Points p and a , may determine the Center C , as it is easie to conceive by the Construction.

Note, That if in this Practice the Lines bp and sc be parallel or make a very acute Angle, the Axis shall not meet with the Plane of the Dial, or else shall meet with it at a great distance from the Point P . Then you must use the following Practice.

Fig. 8. After you have made the same preparation as before, take sg on the Line sc of any length, and from the Point g draw a strait Line gm parallel to sp , which shall be also perpendicular to bp ; from the Point g as a Center, at the distance ga , describe the Arch ae intersecting bp in the Point e .

From the Point A as a Center, and at the distance em describe the Arch MQ , and from the Point P as a Center, at the distance pm , describe the Arch MF intersecting the Arch QM in the Point M , and MPB shall be the Substylar Line.

Then draw the strait Line PZ perpendicular to the Substylar Line MPB and equal to PS the height of the Style, and also MG perpendicular to the same Substylar Line MPB , and equal to mg , the Line GZ shall determine the Position of the Axis in respect of the Substylar Line, and ZE being drawn perpendicular to ZG meeting the Substylar Line in E , the Equinoctial Line shall be

be V E perpendicular to the *Substylar Line* intersecting it in the Point E.

Demonstratson.

There is no difficulty in the Demenstratson of this Practtice after that which has been explained before, we must only conceive here the Triangle e m g to be perpendicular to the Plane of the Dial, that it is Right angled at the Point m, and that it is moved about the Point e which is put upon the Point A, so as the Line e m is always on the Plane of the Dial, and also that the Trapezium p f g m is movable about the Line p f which ought to be applied to the Style P S, so as the Triangle e m g and the Trapezium p f g m may meet in their common Line m g or M G, the Line M G shall be another Style answering to the first, the Point M shall be the Foot thereof, and G shall be the Point, and the Line that passes by the Points of these two Styles S and G, shall be the Axis, and consequently the Line M P that passes by the Feet of these correspondent Styles shall be the Substylar Line.

It is also manifest that the Line Z G determines the Inclination of the Axis in respect of the Substylar Line.

We may take divers Points of Shadow as A, to confirm this Operation, as we have done in the Practtice of the 5th Chapter, the shortest Shadow P B remaining always the same:

It

It is to be observed, That if the *Suns Declination* hath considerably changed between the Observation of the Point of Shadow *A*, and the shortest Shadow *P B*, the Angle *pfc* must be made as we have taught before for the time of the shortest Shadow, and then draw *fa* which makes with *cf* the required Angle for the time of Observation of the Point of Shadow *A*: for it is not necessary that the Observations of the Point of Shadow *A*, and the Point of the shortest Shadow *B*, should be made on the same day.

CH A P. IX.

To find the Center of the Dial, and to draw the Substylar and Equinoctial Lines.

Two Points of Shadow being given, with the Declination of the Sun at the times when the Points of Shadow were observed.

Fig. 9. **A** Style being placed on the Plane of the Dial whereof the Point may be Sand the Foot *P*, and two Points of Shadow *A* and *B* being observed.

Make

Make the *Angle e f a* equal to the Sum or Difference of a *Right Angle* and the Declination of the *Sun* at the hour when we observed the Point of Shadow *A*, following the Rules that we have given in the Practice of the 6th Chapter.

Let several *strait Lines* be drawn, as *A E*, toward the place where we suppose something near they ought to meet the Center of the *Dial*, and from the Center *A* at any distance describe the Arch of a *Circle D R*, intersecting *A E* in the Point *D*. And having made *f a* equal to *S A*, from the Center *Q* at the distance *a r*, equal to *A R* describe the Arch *d r*.

Then from the Point *f* as a Center, at the distance *S D*, describe the Arch *d n*, cutting the Arch *r d* in *d*, and draw the *strait Line a d e* to meet with the *Line f e* in the Point *e*, and make *A E* equal to *a e*, and having found several Points as *E*, draw the *Curve Line E C* by all those Points. If we perform the same Operation by the Point of Shadow *B*, we shall find another *Curve Line F C*, which intersecting the first *Curve Line* in the Point *C*, determines the Center of the *Dial*.

And *C P* shall be the Substylar Line.

We may draw the Equinoctial by the Practice of the 11th Chapter.

The Demonstration of this Practice is so easie, that it needs no Explication.

For we may clearly see that the Intersection *E* of the two *Curve Lines* is the Intersection of the
fides

sides of two Angles $a se$ and $b se$, which are con-
joyned by their common Line se , and there-
fore the sides as and bs on which these Angles
move, are applied to AS and BS , and the
common side of these Angles se which is the
Axis, being prolonged, meets with the *Plane* of
the *Dial* in C , which shall be the *Center* there-
of.

CHAP. X.

*To find the Center of the Dial, and to
draw the Substylar and Equinoctial
Lines.*

*Any Two Points of Shadow being given,
with the Declination of the Sun at
the time of Observation of the Points
of Shadow.*

Fig. 10. **L** Et PS be a Style, whereof P may
be the Foot and S the Point, and
 A and B two Points of Shadow. From the
Point A , at the Distance AS , describe the Arch
 QZ : Also from the Point B , at the Distance
 BS , describe the Arch RZ , intersecting the
Arch QZ in the Point Z , and draw the strait
Lines

Lines, ZB and ZA , and the Line ZP forth at length, then make ZH equal to ZB , and draw the straight Line BH , which being divided into two equal parts in E , let ZE be drawn to meet with AB in the Point F .

Then erect PN perpendicular to ZL , and equal to the height of the *Style* PS , and draw GO and OL at *Right Angles*, and OL shall meet with ZPL at the Point L ; the Line LF shall be one Line which ought to pass by the Center of the *Dial*.

If the Center of the *Dial* that is to be found, be more North than the Point of the *Style*; and if the *Declination* be more North, having made the *Angle* $c\hat{s}b$ equal to the Sum of a *Right Angle*, and of the *Declination* of the Sun at the time when we have marked the Points of *Shadow*; but if the *Declination* be South, having made the *Angle* $c\hat{s}b$ equal to the difference of those Two *Angles*.

And if the Center of the *Dial* required be more South than the Point of the *Style*, and the *Declination* be North, having made the *Angle* $c\hat{s}b$ equal to the Difference of a *Right Angle* and *Declination* of the Sun, but if the *Declination* be South, having made that *Angle* equal to the Sum of the two *Angles*.

Let $\hat{s}b$ be made equal to SB , and from the Point b having drawn bd perpendicular to $c\hat{s}$ prolonged, if it be necessary: from the Point B , and at the Distance bd describe the Arch D cutting ZF in D , then making de equal to DE , draw the straight Line se .

From

From the *Point P* draw *P I N* perpendicular to *F L*, and make *I N* equal to *I S*, and draw *F N*, then draw *N C*, making an Angle with *F N* equal to the Angle *e f c*; the Line *N C* shall meet with the Line *F L*, at the *Point C*, the *Center* of the *Dial* if it have a *Center*, and the same Line *C N* determines the inclination of the Axis upon the Line *C F* on a *Plane* which is inclined to the *Plane* of the *Dial* towards *P*, with an Angle equal to *S I P*.

And *C P* shall be the *Substylar Line*, the *Equinoctial Line* may be found by the *Practice* of the 11th Chapter.

But if the *Dial* have no *Center*, which is known by the Line *N C* being parallel to the Line *F L*; or if the *Center* be at a great Distance from the *Point P*: Then draw the Line *T V X* parallel to *P N*, and make *V X* equal to *V T*, and draw *N X* cutting *V I* in *M*, and draw *P M* meeting *T V* in *Y*, and make *V y* equal to *V Y*, and *y P* shall be the *Substylar Line*. The *Equinoctial Line* is drawn by the *Practice* of the 11th Chapter.

To find the Center by three Points of Shadow.

Having marked three *Points of Shadow*, we may find the Line *F L* by two of those *Points* which shall pass thro the *Center*, and by one of those *Points* and the third, we may find also another Line which shall pass thro the *Center*; and the

Point where these two *Lines* do meet, shall be the Center of the *Dial*.

This Practice may be performed without knowing the Declination of the *Sun*.

The Center being found, and consequently the Substylar *Line* which passes thro the Center and by the Point *P*, the Equinoctial *Line* is drawn by the Practice of the 11th Chapter.

Demonstration.

Fig. 11. To demonstrate this Practice, we must suppose that the Triangle *A S B* in the 11th Figure to be the same with the Triangle *A Z B* of the 10th Figure, so as the Point *Z* is joynd to the Point of the Style, and that the *Line A B* remains in its position.

It is evident that all the *Planes* that are perpendicular to the Plane of the Triangle *S A B* and which passes by the Point of the Style *S*, intersect one another in the *Line S L*, which is raised perpendicular to the Point *S* upon the Plane of that Triangle: Therefore the Plane *L S F* is perpendicular to the Plane *S A B*, and I say that the *Axis* of the *Dial*, and consequently the Center, shall be in that Plane *L S F*.

For the *Line S F* divides the Angle *A S B* into two equal parts; and *S B* being equal to *S H*, the two Triangles *S R D*, *S H D*, being equal each and like to the Triangle *s b d* of the 10th Figure, and being joynd together by their common side *S D*, that *Line S D* shall be the *Axis* in its true position, and the Arch of a Circle *B H*

de-

described from the Center D at the Distance DB, shall be the parallel of the Sun, for the Angle SBD is equal to the Suns Declination.

But in these two *Equicrural Triangles* HSB, HDB, because that the common Base BH is equally divided in the Point E, the Plane that passes by the Lines ES, ED, shall be perpendicular to the Planes of the two Triangles, therefore the Plane which is perpendicular to the Planes of the two Triangles, and that passes by their Vertical Points S, D, shall carry the Axis: but the Plane that passes by the Point S, and by the Point E, and which shall be perpendicular to the Plane of the Triangle HSB, shall be also perpendicular to the Plane of the other Triangle.

Therefore the *Axis* shall be in the Plane FLS that is perpendicular to the Plane of the Triangle ASB, and which passes by the straight Line SEF, and consequently the *Axis* meets with the straight Line FL in some Point, or is parallel to it, and the Point G where the *Axis* meets with the Line FL shall be the Center of the Dial, which was to be demonstrated.

Then to demonstrate the Practice which serves to find the Center of the Dial, we must consider that BH being the Chord of the Arch of the parallel of the Suns Declination contained between the two Lines of Shadow SA, SB, and the Line BD being equal to the Line *bd*, and *de* being equal to DE, the Triangle *sde* is the same with the Triangle SDE of the 11th Figure.

Therefore the *Angle cfe* is that which the *Axis* ought to make with the Line *FL* on a *Plane* which passes by that Line *FL*, and by *S* the Point of the *Style*.

Therefore in the 10th Figure the *Triangle FCN* having the Line *IN* equal to *IS* of the *Triangle FSC*, and being both of them perpendicular to the common Line *IL*, and by the same Point *I*, and the Angle *ENC* being equal to the Angle *esc*, the Point *c* shall be the *Center* of the *Dial*, if the Line *NC* meet with the Line *IL*, which was that which was at last to be demonstrated. We suppose that in this Practice the Difference of Declination between the Observations is not considerably changed.

As concerning the Practice which serves to find the *Substyler Line*, when the Point *C* is at a great Distance from the Point *P*, or when the *Dial* has no *Center*; it is evident by Construction that *TV* being to *VY* as *NI* is to *IP*, the Lines *NT* and *Py* ought to meet one another in the same Point *C*, which shall be the *Center*; or they shall be parallel to one another, and then the *Dial* shall have no *Center*.

CHAP. XI.

To find the Center of the Dial, and to draw the Equinoctial Line.

The Substylar Line being drawn, and one Point of Shadow being given, with the Declination of the Sun.

A N D

To draw the Equinoctial Line the Center of the Dial being found.

A N D

To find the Center of the Dial, the Equinoctial Line being drawn.

Fig. 12. **L** Et a *Style* be placed on the *Plane* of the *Dial*, whose *Foot* let be *P*, and *Point S*, and *Point of Shadow A*, and *Substylar Line C P*.

Make the *Angle dfa* equal to the *Sum* or *Difference* of a *Right Angle*, and of the *Suns Declination*, as we have done in the *Practice* of the *6th Chapter*, *fa* being made equal to *SA*, take any *Point* as *d* upon the *Line fd*, and draw the *Strait Line ad*.

D 4

From

From the Point A draw the strait Line AR perpendicular to the Substylar Line CP, and from the same Point A as a Center, and at the Distance $a d$, describe the Arch N cutting the Substylar Line in N.

From the Point R as a Center, and at the Distance RN, describe the Arch ND; erect the perpendicular PZ at *Right Angles* to the Substylar Line, and equal to PS the height of the Style, then from the Point Z as a Center at the Distance $s d$, describe the Arch GD cutting the Arch ND in D: The Line ZD determines the situation of the *Axis* in respect of the Substylar Line, and if it meets with the Substylar Line as at the Point C, that Point C shall be the Center of the *Dial*.

Then from the Point Z draw the strait Line ZE perpendicular to ZD, meeting the Substylar Line in E, the Line VE perpendicular to the Substylar Line drawn through the Point E, shall be the Equinoctial Line.

If the Center of the Dial were given, the Line CP shall be the Substylar Line, and PZ being perpendicular to the Substylar Line, and equal to the height of the Style, having drawn CZ and ZE perpendicular to CZ, meeting the Substylar Line in E, the Line VE being drawn perpendicular to the Substylar thro the Point E, shall be the Equinoctial Line.

If the Equinoctial Line were given, draw the strait Line PE by the Foot of the Style perpen-

pendicular to the Equinoctial Line VE , and PE shall be the Substylar Line; and having made PZ perpendicular to the Substylar Line PE drawn by P the Foot of the Style, and equal to the height of the Style, having drawn ZE and ZD perpendicular to ZE , the Line ZD determines the inclination or position of the *Axis* with the Substylar Line; and if ZD meets the Substylar Line as at C , that Point C shall be the Center of the *Dial*.

The Demonstration of these Three Practices has nothing in it that deserves to be explain'd after the fore-going Demonstrations; for we may sufficiently know that the Triangle $E Z C$ is the same with ESC which was upon the Plane, which was drawn by the Point of the Style, and by the Substylar Line.

CHAP.

CHAP. XII.

To draw the Equinoctial Line, and the Substylar Line, and to find the Center of the Dial.

Any Two Points of Shadow being given, with the Declination of the Sun.

Fig. 13. **L** Et a *Style* be set on the *Plane* of the *Dial*, whose Foot may be *P*, and Point *S*, and let the Two Points of Shadow be *A* and *B*.

Make the *Angle ass* equal to the Declination of the Sun at the time when we have marked the Point of Shadow *A*, and make *sa* equal to *SA*, and on the Point *A* as a Center describe the Arch *ML* at any Distance; and on the Point *a* describe the Arch *lm* equal to the Arch *LM*.

Then by the Point *A* draw several strait Lines as *AL*, *AM*, toward the place where the Equinoctial ought to pass, which may be known by the Declination being North or South, and knowing near the position of the Wall in regard of the *Axis*.

Then

Then from the Point s as a Center, and to the Distance SM , describe the Arch m cutting the Circle lm in m ; then having drawn am meeting ss in f , make AF equal to af : Also make sl equal to SL , having drawn al cutting ss in d , make AD equal to ad , and so of all the other Lines which we have drawn by the Point A , by which we find the Points as $F D$, and by all those Points as $F D$ we draw a Curve Line $F D$.

We do the same thing for the Point of Shadow B , about which we draw a Curve Line IG , and the strait Line DC , which touches the two Curves, shall be the *Equinoctial*.

By the Practice of the afore-going Chapter we draw the Substyle, and find the Center of the *Dial* (if it has a Center) with the inclination of the *Axis* to the Substylar Line.

Demonstration.

The Demonstration of this Practice is not difficult to those who understand the Conique Sections; they will presently conceive that these Curve Lines, which are traced about by the Points of Shadow, are the Sections of an upright Cone, whereof the Angle at the Vertex of the Triangle drawn by the Axis is double the Declination of the Sun.

The

The Axis of the Cone is a strait Line drawn from the Point of the Style to the Point of Shadow.

Then if we conceive that there is a Plane which touches these two Cones both together, and which passes by their Vertex, as is the Point of the Style; that Plane shall be the Equinoctial; for the Plane that passes by the Axis of the Cone, and by the Line, where the touching Plane meets the Cone, shall be perpendicular to the touching Plane, as may easily be seen; and the Angle contained between the Axis of the Cone and the Tangent Line, being the Declination of the Sun by Construction, the touching Plane shall necessarily be the Plane of the Equinoctial.

CHAP. XIII.

To find the Points of the Hours of 6 and 12 on the Equinoctial Line, and to draw the Meridian Line.

The Equinoctial and Horizontal Line being given.

Or to draw the Meridian, the Center of the Dial being given.

Fig. 14. **L** Et PS be the height of the Style, whereof P is the Foot and S the Point, and let NL be the Horizontal Line, and MN the Equinoctial Line; the Point N where the Equinoctial Line meets with the Horizontal Line, is the Point where the Line of the Hour of 6 intersects the Equinoctial Line.

From the Center N, and at the Distance NS, equal to the height of the Style, describe the Arch KH, and taking any Point as O in the Equinoctial Line for a Center, at the Distance OS describe the Arch IH intersecting the Arch KH in H, then draw the strait Line NH, and HM perpendicular to NH; the Point M where HM meets the Equinoctial Line, is the Point where the Meridian Line ought to intersect the Equinoctial Line.

Then

Then having hanged up a Line with a Plumm^t F , so as the Line may pass by S the Point of the Style , mark any Point as C , on the Plane of the Dial , so as you may see with one Eye the Points M and C both hid together by the Line of the Plumm^t , and this is that which we call bourn^g , and the Line MC shall be the Meridian Line.

But if the Center of the Dial were given , and that it were the Point C, we must mark some Point as M on the Plane of the Dial , which we may see to pass by the Line of the Plumm^t with the Point C , and the Line CM shall be the Meridian.

We may also draw this Meridian Line in the Night with a Candle , in holding it at a distance from the Line of the Plumm^t , so as the Shadow thereof may pass by M or by the Point C which of them is given : For the Shadow of that Line shall be the Meridian Line.

Demonstration.

The Horizontal Circle cuts the Equinoctial Circle into two equal parts and equally distant from the meeting of the Meridian with those Circles; for the Equinoctial is perpendicular to the Axis , and the Horizontal is perpendicular to the Vertical Line , and the Meridian passes by the Axis and by the Vertical Line ; Therefore the Horizontal Line cuts the Equinoctial Line in the same Point where the Hour Circle of 6 meets with them , which is 90 degrees from the Meridian on the same Equinoctial

quinoctial Circle; therefore the Line and Plummert being a part of the Vertical, the Plane that passes by that Line, and by a Point of the Meridian or of the Axis, shall be the Plane of the Meridian, the Intersection whereof with the Plane of the Dial is the Meridian Line.

CHAP. XIV.

To draw the Meridian Line, and to find the Point of the Hour Line of six on the Horizontal Line.

Only one Point of Shadow being given, with the height of the Pole, and the Declination of the Sun.

Fig. 15. **A** *Style* being given, whereof let **S** be the Point, **P** the Foot, being placed on the *Plane* of the *Dial*, draw the Horizontal Line **Hb**, and from the Point **P** draw **PH** perpendicular to **Hb**, and draw **PZ** parallel to **Hb**, and equal to **PS** the height of the *Style*, and make **HE** equal to **HZ**.

Then having mark'd the Point of Shadow **A**, as far from Noon as it is possible, hang a Plummert **T**, so as the Line thereof may pass by **S** the Point of the *Style*, and bourn it (as we did in the fore-going Practice for the Meridian) mark the

the Point *b* on the Horizontal Line, by which you see the Line pass, then when it also passes by the Point of Shadow *A*, and then draw the straight Line *bE*.

Then draw an Arch of a Circle *zof* on the Center *c* at any Distance, and make *zo* equal to the height of the Pole above the Horizon, and draw the Lines *oc*, *zc*, to *c* the Center of the Circle.

Let the Arch *om* be made equal to the Declination of the *Sun* at the time when the Point of Shadow was made toward *Z*, if the *Sun* be in North Signs, and toward *f* if it be in South Signs, for those that have their *Zenith* in the North part of the Sphere; but on the contrary for those that have it in the South part; and draw *am* parallel to *oc*.

Draw *cf* perpendicular to *cz* from the Center *c*, and make the Angle *dcf* equal to the Angle *bSA*, and draw the Line *de* parallel to *fc*, meeting *cz* in *e* and *am* in *a*.

On the Point *E* as a Center, at the Distance *de*, describe the Circle *BD*, meeting *Eb* (prolonged if it be necessary) at the Point *B* make *BM* equal to *da*, and from *M* raise *DM* perpendicular to *EB*, intersecting the Circle *BD* in *D*, then draw *ED* (prolonged if it be necessary) and the Point *F* where it intersects the Horizontal Line, shall be the Point of the Meridian upon the Horizon.

And

And *E G* being drawn perpendicular to *E D*, gives the Point *G* where the Hour Line of six meets with the *Horizontal Line*.

It must be observed that the Line *M D* which is drawn perpendicular to *B E*, may meet with the Circle *B D* on either side of the Point *B*; but you must take care that if the Point of Shadow *A*, is mark'd before Noon, to make use of the Point *D*, which is on the right hand of the Point *B* as in the *Example*, and if the Point *A* were marked after Noon, you must take the Point *D* where *M D* meets the Circle on the left hand of *B* to have the position of the Meridian Line; if *D E* meets not the *Horizontal Line*, but then when it is prolonged toward *E*, the Point *F* shall appertain to the Line of Midnight: All this ought to be understood concerning those that have their Zenith on the North side of the *Equinoctial*, for it is contrary with those which have their Zenith in the *Southern Hemisphere*.

If the Line *E D* meets not the *Horizontal Line* being likewise prolonged towards *E*, then the *Dial* shall have no Line of Mid-day nor of Mid-night, and the Plane of the *Dial* shall be either Oriental or Occidental.

It is also to be observed that the Angle *H E F* made by the Line *E D* with the *Horizontal Line* *E H*, is the Angle of the Declination of the Plane.

By the fore-going Practice the Meridian Line or the Line of Mid-night may be drawn by the Point *F*.

E

De-

Demonstration.

The Figure $f m z c$ is a Projection of the Sphere on a Plane by Lines parallel to one another, which Projection is called the Analemma; the Circle $f m z$ represents the Meridian, and the Point Z the Zenith, $o c$ the Equinoctial Line, $a m$ the parallel of the Sun's Declination at the time when the Point of Shadow A was mark'd, $c f$ the Horizontal Line, $d a e$ the parallel of the Sun's Altitude at the same time, and the Point a determines in the Sphere the place where then the Sun was.

If a Verticle Circle $z a g$ be conceived to pass through the Point a to the Horizon in g , it is manifest that the Line $f g$ in the Circle whose Radius is $f c$, shall be the Versed Sine of the Angle which the Verticle Circle $z a g$ makes with the Meridian Circle $a d f$, and that $d a$ shall be the same Versed Sine in the Circle, whose Radius is $d e$.

Therefore if we conceived the Plane $h E F D$ to be the Plane of the Horizontal Circle, we may see by Construction that the Line $E F D$ is the Meridian on that Plane, and that the Point F is in the Plane of the Dial, the Point of the Meridian where it meets with the Horizon, and that the Point G is the Point where the Hour of six cuts the Horizon, seeing that $F E G$ is a Right Angle; for $B M$ equal to $d a$ is the Versed Sine of the Angle $B E D$, which ought by consequence to be equal to the Angle which is made by the Meridian with the Verticle Circle.

CHAP.

CHAP. XV.

To draw the Meridian Line.

Two Points of Shadow being given in a certain condition.

Fig. 16. **H**AVING fixt a *Style* on the *Plane* of the *Dial*, whereof let *P* be the *Foot* and *S* the *Point*, draw the *Horizontal Line* *Hb* by the *Fourth Chapter*, and from the *Point P* draw *PZ* parallel to *bH*, and equal to *PS* the height of the *Style*, and having drawn the *strait Line* *ZH*, draw *ZC* perpendicular to it.

Then the *Point of Shadow A* being mark'd 3 or 4 hours before Noon, to the intent that the Operation may be more exact, let *AB* be drawn perpendicular to *ZC*.

Then make *BE* equal to *BC*, and *CD* equal to *EA*, and draw *ZD*.

By the place where you judge something near that the *Shadow* of the *Point S* will arrive when the *Sun* shall be so far from Noon as it was when the *Point of Shadow A* was mark'd, draw many *strait Lines* as *QF* perpendicular to *HP*, and by the *Points F* where they intersect *HP*, draw *IFR* perpendicular to *ZC*, and making *FG* equal to *FR*,

E 2

from

from the Point *G* as a Center, and at the Distance *R I* describe an Arch *Q*, which shall cut *F Q* in the Point *Q*, and so we may find as many Points *Q* as we will; and by all these Points drawing a Curve Line *Q Q*, we mark the second Point of Shadow *O*, then when the Shadow of the Point of the Style *S* meets with the Curve *Q Q*.

Then draw the Line *A O*, and on the Point *A* as a Center, and at the distance *A S* describe the Arch *X Y*, also from the Point *O* at the distance *O S* describe the Arch *X N*; cutting the Arch *X Y* in *X*, and draw *X A*, *X O*; then divide the Angle *A X O* into two equal parts by the Line *X M*, which shall meet with *A O* in *M*, the Point *M* shall be one of the Points of the Meridian Line, and by the Practice of the 13th Chapter we may draw the Meridian Line, which ought to pass by the Point *M*.

We suppose in this Practice that the Sun's Declination has not changed considerably between the two Observations of the Points of Shadow *A* and *O*.

Demonstration.

The Curve Line *O Q Q* which ought to pass by the Point *V*, as is easie to understand by the Description, shall also pass by the Point *A*, if we make the Description thereof on the other side of the Line *P H*; and it is the Section of a Right Cone,

Cone, whereof the Vertex is the Point S , and DZC the half of the Triangle which passes by the Axis, couch'd upon the Plane of the Dial, and whereof ZC represents the Axis, the Angle DZC is the Complement of the height of the Sun above the Horizon at the time when the Point A was mark'd, and the Lines as IR are the Sections of the Planes parallel to the Base of the Cone with its Triangle by the Axis, and upon which the Section of the Cone is a Circle, and the Intersections of those Circles with the Plane of the Dial are the Points Q , which are in the Superficies of the Cone and consequently in the Conique Section $AVQQ$ whereof HVP is the Axis: But this Conique Section represents the parallel to the Horizon where the Sun was found at the time of the two Observations of the Shadow in A and O ; Therefore the Line XM which divides the Angle AXO into two equal parts, is the Intersection of the Meridian with the Plane AOS , for that Line shall divide the Chord of that Angle; the Point M is therefore one of the Points of the Meridian Line on the Plane of the Dial, in supposing the Declination of the Sun to be the same for the two Points of Shadow A and O .

CHAP. XVI.

To find the Center of the Dial or determine the Inclination the Axis with the Meridian, to draw the Substylar and Equinoctial Lines.

The Meridian being found; and the height of the Pole being given.

Fig. 17. **L** Et the Point P be the Foot of the Style whereof S is the Point, and let MT be the Meridian. Draw the Horizontal Line Hb intersecting the Meridian Line in T; and from the Point T as a Center at the distance TS describe the Arch BA, and from any Point of the Meridian as M, at the Distance MS describe the Arch DA intersecting the Arch BA in the Point A, and draw the straight Line AT, then make the Angle TAC equal to the height of the Pole above the Horizon which is here given; if the Line AC meets with the Meridian, as at the Point C, then C shall be the Center of the Dial, and the same Line AC in this Position determines the Inclination of the Axis with the Meridian Line; And cP shall be the Substylar Line,

From

From the Point A draw A E perpendicular to A C, intersecting the Meridian Line in E, and the Line E V drawn thro the Point E perpendicular to the Substylar Line C P, shall be the Equinoctial Line.

But if the *Dial* have no Center, draw the Line R P G at pleasure thro the Point P meeting with the Meridian in G, and with the Line A C in R, then draw another Line O N parallel to R G, meeting the same Lines at the Points N and O; and draw the strait Lines R N and G O intersecting one another in the Point F, and make R p equal to G P, and draw p F to meet with the Line O N in Q; and P Q shall be the Substylar Line, and the Equinoctial Line is found as before.

Demonstration.

By the Construction it is manifest that the Line A T represents the Intersection of the Horizontal Plane on the Plane of the Meridian, therefore the Angle T A C being equal to the height of the Pole above the Horizon, it is manifest that the Line A C determines the Inclination of the Axis with the Meridian M T; therefore consequently the Point C shall be the Center of the Dial if the Line A C meets with the Meridian Line: But the Line A E which is perpendicular to A C which

represents the Axis, shall be the Intersection of the Plane of the Equator and Meridian; therefore the Point E shall be one of the Points of the Equinoctial Line on the Meridian Line; but the Equinoctial Line ought always to be perpendicular to the Substylar Line, wherefore E V shall be the Equinoctial Line.

If we have not the Center of the Dial, we have drawn R P G and O Q N parallel to one another, and R N, G O, intersecting in F, and p F Q passing thro the same Point F, then by reason of the like Triangles we have R G to O N as R p or G P to Q N; therefore P Q, M T, R O ought to meet in one and the same Point C, which shall be the Center of the Dial, or else they shall be all three parallel to one another, and then the Dial shall have no Center.

CHAP. XVII.

*Remarques and Practices for divers
Abridgments in the Operations of
the fore-going Chapters.*

I.

Fig. 18. **H**AVING found the Substylar Line pe and the Equinoctial Line eu for the Style sp , if you would remove the Substylar and Equinoctial Lines to another place of the Plane of the Dial; the Line PE parallel to pe shall be another Substylar Line, and VE parallel to eu or perpendicular to PE passing thro any Point of the Substylar Line PE , shall be the Equinoctial Line, and we determine by the following Method the Position of a Style for the two Lines PE , EV , whereof the height shall be given of any length, or we will determine the height of a Style whereof the Position shall be given upon the Substylar Line PE .

First, Let the Line AR be given for the height of the Style, which ought to be set for the Substylar and Equinoctial Lines PE , EV .

Make

CHAP. XVI.

To find the Center of the Dial or determine the Inclination the Axis with the Meridian, to draw the Substylar and Equinoctial Lines.

The Meridian being found, and the height of the Pole being given.

Fig. 17. **L** Et the Point P be the Foot of the Style whereof S is the Point, and let MT be the Meridian. Draw the Horizontal Line Hh intersecting the Meridian Line in T; and from the Point T as a Center at the distance TS describe the Arch BA, and from any Point of the Meridian as M, at the Distance MS describe the Arch DA intersecting the Arch BA in the Point A, and draw the strait Line AT, then make the Angle TAC equal to the height of the Pole above the Horizon which is here given; if the Line AC meets with the Meridian, as at the Point C, then C shall be the Center of the Dial, and the same Line AC in this Position determines the Inclination of the Axis with the Meridian Line; And CP shall be the Substylar Line.

From

From the Point A draw A E perpendicular to A C, intersecting the Meridian Line in E, and the Line E V drawn thro the Point E perpendicular to the Substylar Line C P, shall be the Equinoctial Line.

But if the *Dial* have no Center, draw the Line R P G at pleasure thro the Point P meeting with the Meridian in G, and with the Line A C in R, then draw another Line O N parallel to R G, meeting the same Lines at the Points N and O; and draw the strait Lines R N and G O intersecting one another in the Point F, and make R p equal to G P, and draw p F to meet with the Line O N in Q; and P Q shall be the Substylar Line, and the Equinoctial Line is found as before.

Demonstration.

By the Construction it is manifest that the Line A T represents the Intersection of the Horizontal Plane on the Plane of the Meridian, therefore the Angle T A C being equal to the height of the Pole above the Horizon, it is manifest that the Line A C determines the Inclination of the Axis with the Meridian M T; therefore consequently the Point C shall be the Center of the Dial if the Line A C meets with the Meridian Line: But the Line A E which is perpendicular to A C which

represents the Axis, shall be the Intersection of the Plane of the Equator and Meridian; therefore the Point E shall be one of the Points of the Equinoctial Line on the Meridian Line; but the Equinoctial Line ought always to be perpendicular to the Substylar Line, wherefore E V shall be the Equinoctial Line.

If we have not the Center of the Dial, we have drawn R P G and O Q N parallel to one another, and R N, G O, intersecting in F, and p F Q passing thro the same Point F, then by reason of the like Triangles we have R G to O N as R p or G P to Q N; therefore P Q, M T, R O ought to meet in one and the same Point C, which shall be the Center of the Dial, or else they shall be all three parallel to one another, and then the Dial shall have no Center.

CHAP.

CHAP. XVII.

*Remarques and Practices for divers
Abridgments in the Operations of
the fore-going Chapters.*

I.

Fig. 18. **H**AVING found the Substyler Line pe and the Equinoctial Line eu for the Style sp , if you would remove the Substyler and Equinoctial Lines to another place of the Plane of the Dial; the Line PE parallel to pe shall be another Substyler Line, and VE parallel to eu or perpendicular to PE passing thro any Point of the Substyler Line PE , shall be the Equinoctial Line, and we determine by the following Method the Position of a Style for the two Lines PE , EV , whereof the height shall be given of any length, or we will determine the height of a Style whereof the Position shall be given upon the Substyler Line PE .

First, Let the Line AR be given for the height of the Style, which ought to be set for the Substyler and Equinoctial Lines PE , EV .

Make

Make EP equal to ep , and set it the same way (that is the Point P must be set above the Point E if the Point p be above the Point e , and below it, if it be below it) and make EZ equal to ps , and EZ equal to RA given, and draw zP and ZR parallel to ZP , meeting EP in R ; and the Point R shall be the Foot of the Style, the height whereof RA is given.

Therefore if you fix a Style, whereof the Foot may be R , and the Distance between the Point thereof A and Foot R , may be the height equal to the Line ZE , the Proposition is satisfied.

But if the Point R were given for the Foot of the Style, and the height were required: draw PZ as before, and by the Point R draw RZ parallel to Pz , and EZ shall be the height of the Style whose Foot is the given Point R .

II.

The Substylar Line CE , the Equinoctial Line EV , and the Meridian Line CM , answerable to the Style SP being given, we may take what Point we will in the Substylar Line as K , to the Center of the Dial, without altering the Substylar Line or Equinoctial, and the Line Km drawn parallel to CM shall be the Meridian Line, but the height and position of the Style must be changed by the following Method.

Make

Make Ez equal to PS , and draw MP and PZ , and draw mR and RZ parallels to MP and PZ , and the Point R shall be on the Substylar Line, which is the Foot of the Style, whereof the height is RZ perpendicular to the Point R on the Plane of the *Dial*.

III.

If the Substylar Line CE were given with the Meridian Line CM answering to the Style PS , we may take any Point as p to be the Foot of a Style, whereof the height is to be determined; or the Style being given of any height to determine the Position of the Foot p without changing either the Meridian or the Center of the *Dial*.

If the Foot of the Style p be given, and we are to determine its height, by the Foot of the Style P , put for the finding of the Meridian and Substylar Line, let there be drawn PS perpendicular to the Substylar Line and equal to the height of the same Style, and let CS be drawn by the Center of the *Dial*; and from the given Point p let ps be drawn parallel to PS , till it meet CS in the Point s , and ps shall be the length of the height of the Style, which ought to be placed at the Point p , and the Meridian CM and the Center of the *Dial* C are not changed.

But

But if ps were given for the height of the *Style*, it must be put upon PS prolonged if it be necessary in Pz , and $z s$ must be drawn parallel to CP to meet with the Line CS in ms , and sp being drawn parallel to SP , shall give the Point p on the Substylar Line for the Foot of the *Style* required, whereof the height is given.

IV.

Fig. 19. The Meridian CM being given, with the Equinoctial Line EM , we may find another Equinoctial as em , without changing the *Meridian*, the which Equinoctial em shall make the Angle emC with the Meridian CM equal to the Angle EMC , but we must find another *Style* by the following Method:

If the *Center* of the *Dial* be given at the Point C , by the Foot of the *Style* P which have served to find the Meridian and the *Center* C , having drawn PS perpendicular to the Substylar Line CP , and equal in length to the same *Style*, let es be drawn parallel to ES , and from the Point s draw sp parallel to SP , meeting the Substylar Line in the Point p , which shall be the Foot of the *Style* required, whereof ps shall be the height.

But if we have not the *Center* of the *Dial*, we must draw the Line Ss by the Point S which determines the Inclination of the *Axis* with the Sub-

Substylar Line, by the Practice of the 16th Chapter, and we shall find as before the Point p for the Foot of the *Style* required, whereof the height shall be $p f$.

V.

Fig. 20. If after you have drawn the *Meridian Line* $C M$ and the *Substylar Line* $B P$, we cannot have the *Equinoctial*, because the *Style* has been put too long, we may diminish it as much as we please, without changing either the Foot thereof or the *Substylar Line*; but we must find another *Meridian* and another *Horizontal Line* which may answer to that *Style*, and these *Meridian* and *Horizontal Lines* shall be parallel to the first *Meridian* and *Horizontal Lines*.

Therefore draw the Line $e m M$ by any Point of the *Substylar Line* as e , which may be perpendicular to it, that Line may be the *Equinoctial Line*; but the height of the *Style* must be changed in drawing $e f$ perpendicular to the Line $S f$ which determines the inclination of the *Axis* with the *Substyle*, and that Line $e f$ meeting $S P$ which is a perpendicular to the *Substylar Line* by the Foot of the *Style*, and which is his height, so that for the *Equinoctial Line* $e m$, $P z$ shall be the height of the *Style* required; but there must be another *Meridian* found, whether the *Center* of the *Dial* be found or not found.

Take

Take on the *Substyle* the length eB equal to e , draw BM , and also take eb on the *Substyle* Line equal to $e z$, and by the Point b draw bm parallel to BM meeting the *Equinoctial Line* em in the Point m , and let cm be drawn parallel to CM , the Line cm shall be the *Meridian* for the *Style*, whereof the Foot is in P , the height is Pz , and the *Equinoctial Line* em .

And we may yet change if we will the Foot of the *Style* or its height, according as necessity requires by the former Observations.

But for the *Horizontal Line* DH which has been found for the *Style* whose Foot was P and PS the height, the which meets the *Substyle* at the Point H , if you would find another for the *Style* whose height is Pz , without placing effectually that *Style* to make use of the *Practice* taught in the 4th Chapter, you have no more to do but to draw SH , and by the Point z the Line Zb parallel to SH meeting the *Substyle* in b , and the Line db parallel to DH , shall be the *Horizontal Line* required.

VI.

Lastly, a *Dial* being drawn on a *Plane*, we may transfer it in what other place we will on the same *Plane*, by drawing of parallel Lines to those that are drawn, so that we keep the same order and the same proportion between them in their meetings, but the *Style* ought to be put at the Point which answers to the Point of the first, which is for its Foot.

THE

THE
SECOND PART.

THE
P R E F A C E.

Concerning the choice which we ought to make of the Practices to draw the Substylar, Equinoctial, and Meridian Lines ; and to place the Center of the Dial, according to the Expositions of the Planes proposed.

HAVING taught in the first Part of this Treatise different Practices to find the principal Lines of *Sun Dials*, I think it will not be amiss to give some Instructions concerning the Use which we may make of these Practices, following the different Expositions of the *Planes* proposed on which the *Sun Dials* are to be drawn.

Therefore it is necessary to know something near, the Position of the *Plane* in regard of North or South before we begin any thing; which may
be

be done by a small Declinatory, which presently shews on what side the North, South, East, or West is; which those that are used to observe the *Sun*, may know by seeing in what manner it shines upon the Plane according to the Hour and Season of the Year.

Then we may well conceive after what manner the *Axis* shall meet with that Surface, and consequently we may judge of the Position of the *Substylar Line*, of the *Equinoctial*, and also of the whole *Dial*.

But considering a *Dial* wholly made, it is not difficult to know among divers manners which we may use, that may be most fit and most easie for the Construction of the *Dial*.

Therefore we may presently see that it will be useless to find the *Center* of a *Dial* or the *Meridian* on a *Plane* which comes near either to the East or West, and that the *Equinoctial Line* being set on such a *Plane*, we need not find the Point of Mid-day, and that we must use the Point of the Sixth Hour to begin the Divisions of the Horary Intervals on that Line; or on the *Horizontal Line*.

Also we may see that on these sorts of *Planes* we cannot use the Practices where we ought to have the Points of Shadows after Mid-day, which may be answerable to others taken in the Morning; for that which is of those which give the Position of the *Meridian* of the *Plane*, which is the

the Substylar Line, by the correspondent Points, we can never fitly make use of them on these sorts of *Planes*: For if the first Point hath been mark'd a little too far from that Meridian, we can never have it's correspondent Point.

Also we must not use the Practices of correspondent Points of Shadows, or the Tract of the Shadow, if the *Circle* that is described from the Foot of the *Style* as a *Center*, meets that Tract in *Angles* too acute; for we cannot determine exactly *that meeting*, and this inconvenience may happen to the Practices on all sorts of *Planes* in any Season of the Year.

If the *Dial* be large, or if the *Declination* of the *Sun* has changed considerably between the *Observations* of the *Points* of Shadow, we have not exactly the Lines which we seek by those Practices where we suppose that it hath not been changed between the *Observations*.

In the Practices of this *Second Part* we suppose always that the Equinoctial Line or Horizontal Line is drawn, and that we have mark'd on that Line the *Point* where the Hour of 12 or 6 meets with it; at which *Points* we begin the Division of the Hours on those *Lines*; but to draw them we must have the *Center* of the *Dial*, or at least the *Inclination* of the *Axis* to the *Substylar Line*, which has been taught in the *First Part*.

CHAP. I.

To mark the Points of the Astronomique Hours on the Equinoctial Line, and by those Points to draw the Hour Lines.

Fig. 21. **L** Et SP be a *Style*, whereof S is the Point and P the Foot, E XII, is the Equinoctial Line; on which the Point XII is the meeting of the Equinoctial with the Meridian, and the Point VI is the meeting thereof with the Hour of Six and with the Horizon; PEA is the Substylar Line which meets with the Equinoctial in E.

Make EA on the Substyle equal to ES, which is the Distance between the Point E of the Equinoctial and S the Point of the Style, and draw AXII or A VI, or both of them, if you have those two Points on the Equinoctial Line; the which two Lines AXII, A VI, ought to make a Right Angle at the Point A; then on the Point A as a Center, at any Distance describe an Arch of a Circle *bc*, which shall cut the Lines AXII and A VI at the Points *b* and *c*, and divide the Circle from 15 to 15 Degrees, beginning at the

the Point *b*, or at the Point *C*; then draw strait Lines from the Center *A*, and by the Points of the Division of the Circle, which must be prolonged, if it be necessary, to the Equinoctial Line, on which it shall give the Division of the Hours which are to be mark'd, according to the apparent motion of the Sun from East to West.

Then by the Center of the Dial, and by the Points of the Hours which are mark'd upon the Equinoctial Line, draw strait Lines, which shall be the Hour Lines.

But if we have not the Center of the Dial, and we have only the Inclination of the Axis *Zz* to the Substylar Line *eE*, you must take any Point, as *e*, on the Substylar Line, and by that Point *e* having drawn a strait Line *e 12*, parallel to the Equinoctial Line *E XII*; and drawing *e z*, perpendicular to *Zz*, make *ea* equal to *ez*, and by the Point *a* draw the strait Lines *a 11*, *a 12*, *a 1*, &c. parallel to the Lines *A XI*, *A XII*, *A I*, &c. and by these Points, where these Lines meets with the Line *e 12*, and by those which are correspondent to them on the Equinoctial Line, draw the Hour Lines *11 XI*, *12 XII*, *1 I*, *2 II*, &c.

Demonstration.

The Plane *S XII E* is the Plane of the Equinoctial by the Constructions of the First Part, and the Triangle *A XII E* being the same with the Triangle *S XII E*, we ought to put upon the Plane of

E z the

the Triangle AXI the Hour Lines 15 Degrees the one from the other about the Point A , in beginning from the Line AXI or AVI , which are the Intersections of the Plane of the Meridian, or of the Plane of the Hour Circle with the Plane of the Equinoctial; but all the Planes of the Hour Circles intersect one another in the Axis, and do also pass by the Center of the Dial which is the meeting of the Axis with the Plane of the Dial: Therefore the straight Lines drawn from the Center of the Dial to the Points of the Hours mark'd on the Equinoctial Line shall be the Hour Lines of the Dial, if the Dial has no Center; or if the Center be at a great distance from the Equinoctial Line, the Practice that is here taught for that Case gives the Points on the Line $e12$, which is parallel to the Equinoctial Line; so as all the distances of the Hour Lines on that Line $e12$, are in the same proportion with their Distances on the Equinoctial Line, and that each of these Distances on the Line $e12$ have the same proportion to their correspondent Distance on the Equinoctial Line, as the Line $e12$ has to the Line EZ ; therefore it follows that if the Dial have no Center all the Hour Lines are parallel to one another and to the Axis. And if it have a Center, all the Lines shall meet in that Center which is very manifest, because of the like Triangles which are made by the Lines $E12$, $e12$ which are parallel to one another.

CHAP. II.

*To mark the Points of the Astronomique
Hours on the Horizontal Line.
And to draw the Hour Line by those
Points.*

Fig. 22. **S** Is the Point of a *Style* given, where-
of **P** is the Foot, **MHD** is the Ho-
rizontal Line, **PH** is drawn by the *Poine* **P** (the
Foot of the *Style*) perpendicular to the **Horizon-
tal Line**; **M** is that Point where the Meridian
Line intersects the Horizontal Line, upon the
Line **HP** set **Hf** equal to **HS**, and draw the
strait Line **fMa**, and make the Angle **MfA**
equal to the Angle of the elevation of the *Pole* a-
bove the *Horizon*, then from any Point as **A**, ta-
ken on the Line **fA**, raise a perpendicular from
12 to **fA**, till it meet with **fM** in **12**, and draw
the Line **9, 12, 4**, perpendicular to **fa**, and
make **12a** equal to **12A**; and from the Point **a**,
as a Center, describe a Circle at any Distance, and
divide it into equal parts from **15 Degrees** to **15
Degrees**, beginning the Division where the
Line **a 12** intersects the Circle; and draw Lines
from

from the *Point a* , to the Divisions of the Circle, to meet with the Line 9.4 at the Points 9, 10, 11, 12, 1, 2, 3, 4, &c. And by the same Points and the Point *f*, draw strait Lines which shall meet the Horizontal Line in the Points of the Hours required , which must be mark'd according to the *Diurnal Motion* of the Sun , of which the Point *M* shall be Noon , and *D* the Point of the Hour of Six.

If the Line *fD* drawn perpendicular to *fM* meet with the *Horizontal Line* at the Point *D*, that Point shall be the Hour of Six on the *Horizontal Line* , which is the same Point where the *Equinoctial Line* ought to meet with the *Horizontal Line* , as hath been taught in the *First Part*.

If we have not the Point of Mid-day on the *Horizontal Line* , and we have but *D* the Point of the Hour of Six , then draw *fD* and *fM* perpendicular to *fD* , the which *fM* meets the *Horizontal Line* , or not in *M* , for it is indifferent ; then we do the same as we did before to find the Points of the Hours on the *Horizontal Line*. We may see by this *Practice* that it is not necessary that the Line *fM* should meet with the *Horizontal Line*.

The Hour Lines are to be drawn from *C* the Center of the *Dial* , and by the Points of the Hours which have been found on the *Horizontal Line*.

If the *Dial* have no Center , it is better to follow the Method of the precedent Chapter than this,

this, seeing that we have always an Equinoctial Line in that Case.

Demonstration,

The Plane fMD is the same with the Plane SMD by Construction, which is the Plane of the Horizon; therefore all this Operation must be considered as made on the Plane of the Horizon which passes by the Point of the Style S , whereof fM is the Meridian given, or the Substylar Line, or fD is the Line of the Hour of Six, the Point f is the Center of the Dial, the Line fA is the Inclination of the Axis to the Substylar Line fM ; therefore it is that taking the Point A at pleasure for the Point of the Style, by the Practice of the 11th Chapter the Line 9, 4 , shall be the Equinoctial answering to the Point of the Style, and by the Practice of the former Chapter the Lines f12 , f1 , f2 , &c. shall be the Hour Lines on that Plane, which meets the Horizontal Line MHD , which is also on the same Plane by Construction with the Points of the Hours which they design; therefore the Lines which are drawn thro those Points of the Hours on the Horizontal Line, and by C the Center of the Dial, shall be the Hour Lines of the proposed Plane.

CHAP. III.

Six Intervals of Hours following one another being given, to draw all the other Hours.

Fig. 23. **A**S let the Six Intervals of Hours from *C* to *C* be given, draw *Ee* parallel to *cs*, cutting *cA* in the Point *A*, *CB* in the Point *B*, *CD* in the Point *D*, &c. And make *Ab* equal to *AB*, *Ad* equal to *AD*, &c. And from the Center *C*, and through the Points *bde*, &c. draw the Lines of the Hours that follow the precedent Hours.

If also you would have other Hours following the first or last found, you must repeat the Operation in drawing another Line as *Ee* parallel to that which is the last of the Six Intervals of Hours.

If the *Dial* has no Center, you must draw another Line as *ft* parallel to *Ee*, on which find the Points of the Hours as you found them on the Line *Ee*, and in joyning the *Horary Points* of the two parallel Lines *Ee* and *ft*, you shall have the Hour Lines required.

Demon-

Demonstration.

If we imagine a Plane to pass by the Line $E A e$, and which is parallel to the Plane of the Hour Circle of the Line $C 5$, it is manifest that the Plane of the Hour Circle which passes by the Line $C A$, shall be perpendicular to the Plane that passes by $E A e$, because that the Planes of the Hour Circles $C A$, $C 5$, are inclined to one another by a Right Angle, and by consequence the Axis meets not with the Plane drawn by $E A e$, and it shall be parallel to it, and all the Lines which are the meetings of the Planes of the Hour Circles with the Plane that passes by $E A e$, shall be all parallel to one another and to the Axis.

But because the Plane that passes by $C A$ is perpendicular to the Plane by $E A e$, the meetings of the Hour Lines upon that last Plane shall be equally distant by order from the meeting of the Plane drawn by $C A$; that is to say, that the first on one side shall be as far distant as the first on the other side, and that the second hour on one side shall be as much as the second hour on the other side, and so following in order; therefore the Line $E A e$ which is on that Plane, meets the parallel Hour Line at equal distance by order, both on the one and the other side of the Point A , which was to be demonstrated.

We

We may conceive the Plane that passes by $E A$ as the Plane of a Dial which has no Center, upon which all the Hour Lines are parallels to one another and to the Axis; and that the Hour Line which passes by the Point A is the Meridian or the Substylar Line of the Plane; then we may see manifestly that the Hour Lines that are on one and the other side of that which passes thro A , are equally distant from it in order, and by consequence if we draw a strait Line any way upon that Plane, the meetings of the Hour Lines both on the one side and on the other of that which passes by A , shall be at an equal Distance from A .

CHAP. IV.

To draw the Parallels of the Twelve Signs.

Fig. 24. First draw the Line sc and sa at Right Angles to it at the Point s , make the Angles asd , ask , each 23 Degrees 30 Minutes, and the Angles ass , asi , each of 20 Degrees 11 Minutes, and the Angles asg , ask , each of 11 Degrees 30 Minutes, the Line sa denotes the Equinoctial, which is the beginning of *Aries* and of *Libra*; the Line sk denotes the be-

beginning of *Taurus* and *Virgo*, *fi* the beginning of *Gemini* and *Leo*, *fk* the beginning of *Cancer*, which is the *Tropick* of the same *Sign*; *fg* the beginning of *Scorpio* and *Pisces*, *ff* the beginning of *Sagittarius* and *Aquarius*, *fd* the beginning of *Capricorn*, which is the *Tropick* of the same *Sign*.

If the *Center* of the *Dial* be towards the *North* in regard to the *Point* of the *Style*, make *sc* equal to *SC* of the *Dial*, which is the distance between the *Point* of the *Style* and the *Center*, but if the *Center* be towards the *South*, in respect of the *Point* of the *Style*, make *sc* upon *cs* prolonged on the other side of the *Point* *s*.

Then to find the *Points* of the *Parallels* of the *Signs* upon the *Hour Lines*; as for *Example*, On the *Line* of *Mid-day*, you must take the distance *SXII* from the *Point* of the *Style* *S* to the *Point* *XII*, which is the *Intersection* of the *Line* of *Mid-day* with the *Equinoctial*, and set it from *s* to *12* upon the *Line* *sa*, and having drawn the *Line* *c12* which cuts the *Lines* of the *Signs* in the *Points* *d, f, g, b, i, k*, then we transport the *Intervals* *12b, 12i, 12k, 12g, 12f, 12d*, in *XIIH, XII I, XII K, XII G, XII F, XII D*, on the one and other side of the *Equinoctial* *Line*, as they are on the two sides of the *Line* *sa*.

And after the same manner having found the other *Points* upon each *Hour Line*, and likewise on the halves and quarters, or other *Lines* coming from

from the *Center* , we draw by all the *Points* which belong to the same *Sign* , the *Line* of the *Parallel* of the *Sign* , and so for each of them in particular.

But if we have not the *Intersection* of the *Equinoctial Line* upon the *Hour Line* , on which you would have the *Points* of the *Signs* , in that case you may have always the *Center* of the *Dial* ; therefore it is, that having taken any *Point* , as *R* , on the *Hour Line* , and having made the *Triangle* cfr on the *Base* cf equal , and like to the *Triangle* CRS on the *Base* CS , and the *Line* cr being continued if it be necessary , shall meet with the *Lines* of the *Signs* in those *Points* , which being set on the *Dial* , in observing from the *Point C* which is the *Center* , the same *Distances* which they have from the *Point c* .

But if we have not the *Center* of the *Dial* , we may always have the *Equinoctial Line* ; therefore, having taken (as for *Example*) the *third Hour* , on which we would have the *Points* of the *Parallels* of the *Signs* , then having taken the *Point R* at pleasure , and having mark'd f_3 on the *Line* sa equal to $SIII$, which is the *distance* between the *Point of the Style S* , and the *Point* where the *Line* of the *third Hour* proposed intersects the *Equinoctial Line* ; on that *Line* f_3 for the *Base* let the *Triangle* f_3r be made equal and like to the *Triangle* $SIII R$, which has $SIII$ for its *Base* ; and draw r_3 prolonged , which shall intersect the *Lines* of the *Signs* in *Points* which are to be trans-

transferred to the Line of the Third Hour, as we have taught here before.

The Construction of this Practice is so plain, that it needs no Demonstration, for it is easie to see that the Plane *cf* 12 is the same with the Plane of the Hour Circle *CSXII*, and so of the rest.

A part of an Arch of a Sign being given, which is Parallel to the Equator, we may describe that Arch by the Practice of the following Chapter.

CHAP. V.

The Equinoctial Line being given, we may draw a Parallel to it by a Point given on an Hour Line.

Fig. 25. **T**HE Equinoctial Line *AG* is given, and the Hour Lines *a A, P B, c C, d D, &c.* which intersect the Equinoctial in the Points *ABCD, &c.* and the Point *P* on one of the Hour Lines is also given, by which it is required to draw a Parallel to the Equator.

By the Point *P*, and by the Point *A*, on the Equinoctial, the Point of the Hour next to *B*, on which is the given Point *P*, draw *AP* meeting with

with the next Hour to PB at the Point c , then draw cE , so as the Points E and A may be equally distant from the Hour cC ; the intersection d of the Line cE with the Hour Line Dd shall be one of the Points of the required Parallel; after the same manner we find another Point f , and so of the rest.

But here it is to be observed, that if there be only whole Hours on the *Dial*, we shall not have the Points of the Parallel but for every other Hour; and if we have the Lines of the half Hours, we shall have the Points of the Parallel from Hour to Hour, and if we have the Lines of the Quarter of Hours, we shall have the Points of the Parallel from Half Hour to Half Hour, and so of the rest; for if AP meet the Line of the half Hour fb at the Point f , having drawn fD so as bD and bA may be equal, fD shall meet the Hour Line cC at the Point K , which is as far from fb as PB , the Point K shall be one of the Points of the Parallel.

If instead of the Equinoctial Line we have one of its Parallels, that parallel being given, we do the same thing as if it were the Equinoctial Line.

Demon-

Demonstration.

The strait Lines on the Plane of the Dial represent great Circles of the Sphere, therefore it is that the two great Circles represented by CA and by CE, which are equally inclined to the Hour Circle c C, because that CA and CE are equal, meeting PB and Dd, which are equally distant and equally inclined to Cc, in the Points P and d, equally distant from the Equinoctial Line BD, therefore P and d shall be in one and the same parallel to the Equator, and so of the other Points.

CHAP. VI.

To draw the Italian and Babilonian Hours upon an Horizontal Plane.

Fig. 26. THE Astronomique Hours being drawn on the Dial whose Center is C, and the Meridian CA and VA the Equinoctial Line, cE being divided into two equal parts in A, let the Points b, c, d, e, f, g, h, &c. of a Parallel to the Equator, be found on the Hour Lines by the Practice of the fore-going Chapter, which Parallel passes by the Point A.

The

The Line A 12 parallel to the Equinoctial, shall be the Line of the 12th *Italian Hour*.

The strait Line b VII, which passes by the Point of the Seventh Hour in the Morning of the Equinoctial, and by the Point of the First Hour after Noon of the Parallel, shall be the Line of the 13th *Italian Hour*.

The strait Line c VIII which passes by the Point of the Eighth Hour in the Morning on the Equinoctial Line, and by the Point of the second Hour after Noon of the Parallel shall be the 14th *Italian Hour*, the strait Line d IX, which passes by the Point of Nine in the Forenoon on the Equinoctial, and by the Point of Three in the Afternoon on the Parallel, shall be the 15th *Italian Hour*, and so of the rest; there being always Six Hours distance between the Hour of the Equinoctial and that of the Parallel.

The *Babylonian Hours* are mark'd after the same manner, but only that which is done on one side of the Meridian for the *Italian Hours*, is made on the other side of the Meridian for the *Babylonian Hours*, and they are counted after another manner; as for *Example*, the strait Line that passes by the Point of Mid-day of the Equator, and by the Point of the Sixth Hour in the Morning of the Parallel, is the Sixth *Babylonian Hour*; that which passes by the first Hour Afternoon on the Equinoctial, and by the Point of the Seventh Hour in the Morning on the Parallel, shall be the Seventh *Babylonian Hour*, and so

so following, so as A 12 parallel to the Equator shall be the 12th *Babylonian Hour* for the *Horizontal Dial*.

Demonstration.

Fig. 27. If we imagine a Parallel to the Equator, which passes thro the Intersection of the Meridian and Equator, it is evident that the Horizontal Circle shall touch that parallel at the Point where the Meridian Circle cuts it; and if we conceive that the Horizontal Circle be fastened with the Parallel and the Meridian, then when the Meridian hath advanced 15 Degrees on the Equator in going according to the Motion from the East to the West, the Plane of the Horizontal Circle in that Position, shall mark the first Italian Hour on the West part, and the first Babylonian Hour on the East part; and then when it hath advanced 15 degrees more, the Plane of the Horizontal Circle shall mark the second Italian and Babylonian Hour, and so on; so as then when it is arrived to the 12th Hour, the Meridian Circle shall be found in its first Position, for it has made half a Revolution; but the Horizon that cuts the Plane of the Meridian at Right Angles, shall be found in a Position opposite to that which it had at first; that is to say, that if S H be the meeting of the Horizon with the Meridian, S C the Axis, C E the Plane of the Dial parallel to the Horizon; after that the Meridian hath passed over 12 Hours of the Equator, or that S H

G

has

has puffed over 12 Hours of the Parallel ; the Line SH shall fall in 12 SA : So as the Angle 12 SC shall be equal to the Angle $HS C$, therefore CSA shall be an Equicrural Triangle ; and so also the Triangle SAE shall be Equicrural, SE being the meeting of the Equator and Meridian ; therefore SA is equal to AC and AE both together ; therefore CA and AE are equal : But CE represents the Meridian of the Horizontal Dial, wherein C is the Center, E the Intersection of the Equinoctial, and A the Intersection of the 12th Italian or Babylonian, which must be perpendicular to the Meridian, for in that Position the Plane of the Meridian is return'd to its place, and the Horizon which has not changed its Inclination which it has with the Meridian remains always at Right Angles to it. Therefore it shall intersect the Plane of the Dial in a Line perpendicular to the Meridian, the Plane of the Dial being parallel to the Horizon in that first Position, and that Line ought to pass thro the Point A of the Meridian, which divides the Line CE into two equal parts.

If we conceive a Cone which has for its Base the Parallel to the Equator $H 12$, and for its Vertex the Center of the Sphere, the Section of that Cone with the Plane of the Dial, shall be the representation of that Parallel, but the Plane of the Dial is parallel to one of the Planes, which touches the Cone ; therefore the Section or the Representation of that Parallel shall be a Parabola.

And

And seeing that all the Planes of the Italian and Babylonian Hours are also the Planes which touch that Cone, the Lines of those Hours shall touch the Parabola which is in that Section.

CHAP. VII.

To draw the Italian and Babylonian Hours, on a Plane which is not Horizontal.

Fig. 28. **T**HE Astronomique Hours being described, and the Horizon RH which is one of the Hours required, being drawn on the Plane of the Dial with the Equinoctial Line, we draw a Parallel to the Equator d, b, R, e , f, g , which passes by R , the Intersection of the Horizon, with any Hour Line, as in this Example, with the Hour Line of II , by the Practice of the 5th Chapter. And seeing that the Horizon, which is the Line of the 24th Italian Hour, intersects the Parallel in R , at the Point of the Second Hour after Noon, and the Equinoctial Line at the Point of the Sixth hour after noon the Line of the first Italian Hour shall pass by the Point e of the Parallel, which is the Third Hour after Noon, and by the Point of the Equinoctial; the Line of the Second Italian Hour shall pass by the Point f of the

Parallel, which is the Fourth Hour, and by the Point of the Eighth Hour on the Equinoctial Line; and so of the rest we find all the Points by which the *Italian Hours* ought to pass, so as the Eighteenth *Italian Hour* passes always by the Point of Mid-day of the Equinoctial Line, and by a Point of the Hour of a Parallel, which shall be so far from the Point of Mid-day as the Point R of the same Parallel, which is the Intersection of it with the Equinoctial Line, is the Point of the Sixth Hour after Noon, which is in this *Example* at Four Hours distance, seeing the Point R is an Hour of the Afternoon.

Fig. 29. But if the Point R, by which the Parallel to the Equator is described, were the Intersection of an Hour before Noon, we must consider that that Parallel ought to meet also the Horizon in a Point of an Hour, which is so far from Noon, as that is by which we have described it; for *Example*, If the Point R were the Intersection of Nine in the Morning with the Horizontal Line, the Parallel to the Equator described by the Point R, ought to meet the Horizontal Line in the Point H, which is upon an Hour Line, so far distant from Noon as is the Point R; that is to say, that the Point H shall be the meeting of the Third Hour after Noon with the Horizontal Line; and the Line of the 24th *Italian Hour*, which is an Occidental Portion of the Horizon, ought to be taken from the Point of the Third Hour of the

the Parallel with the Point of the Sixth Hour after Noon of the Equator, and in counting as we have done before, we shall find that the First Italian Hour shall pass by the Point of the Fourth Hour on the Parallel, and by the Point of the Seventh Hour after Noon on the Equinoctial; and that the Line of the Second Italian Hour shall pass by the Point of the Fifth Hour of the Parallel, and by the Point of the Eighth Hour of the Equinoctial, and so on; and we draw only those that are visible, for the others are of no use, and serve only to count and to place those which are of use.

These Rules are for the Italian Hours, but for the Babylonian Hours, which have for the Twenty fourth Hour the Oriental Part of the Horizon, if the Parallel which is described by the Point R of the Horizon, were the meeting of the Horizon with the Line of the Ninth Hour before Noon, the First Babylonian Hour shall pass by the Point of the 10th Hour in the Morning of the Parallel, and by the Point of the 7th Hour in the Morning on the Equinoctial; the Line of the Second Babylonian Hour shall pass by the Point of Eleven before Noon on the Parallel, and by the Point of Eight on the Equinoctial, and so of the rest; and if the Point R of the Parallel were the Point of any Afternoon Hour, we must take its correspondent before Noon to begin to count the Babylonian Hours, which is the contrary of that which

which we have done for the Italian Hours.

We must observe, that if we describe the Lines by the Intersections of the Astronomique Hour-Lines with the *Italian* or *Babylonian*, in taking those Points of Intersection in order from the Equinoctial Line, those Lines shall be the Parallels to the Equator, which shall meet the Horizontal Line in the same Points where the Hour Lines meet.

Demonstration.

After that which hath been demonstrated in the afore-going Chapter, concerning the generation of the Italian and Babylonian Hours, it suffices here to explain after what manner the Horizontal Line meets the Astronomique and Italian Hour Lines in one and the same Point.

It is easie to see that when the Horizon, which we have supposed to be fastened to the Meridian without changing its Inclination that it has with it, is moved by the motion of the Meridian, that if the Meridian advanceth 15 degrees on the Equator; also that Point of the Horizon, let it be what it will be, shall advance 15 degrees on the Parallel which that Point describes: Therefore if we describe a Parallel to the Equator by one Point of the Horizon, by which Point passes an Astronomique Hour Line, it is manifest that all the same Hour Lines shall divide that Parallel from 15 to 15 degrees; Therefore it is that all the Italian Hour Lines

which

which ought to pass by the Points of the Hours of the Equinoctial, shall also pass by the Points of the Hours of the Parallel; and it is also manifest that we may draw Six Parallels to the Equator by the six Intersections of the Horizon with the six Hour Lines 12, 1, 2, 3, 4, 5; and that the Italian Hours pass in order by the Points of the Hours of those Parallels; and it also follows that all the Lines of the Italian and Babylonian Hours shall touch the Parallel described by the Point of meeting of the Horizon and Meridian, and that all the Points of touching shall be the meeting of the Astronomique Hours with the Parallel, and that there will be always six Hours Interval between the touching Point upon the Parallel, and the Point of meeting of the same Hour upon the Equator; for the Plane of the Horizon touches that Parallel at the Point of Mid-day, the which Point describes the Parallel.

CHAP. VIII.

To continue the Description of the Italian and Babylonian Hours, when the Parallel or the Equator is wanting on the Plane of the Dial.

Fig. 28. IF the Point *b* be the last that is found on the Parallel by means of the Equinoctial Line, following the Practice of the 5th Chapter of this Second Part, and that the Line *b III* be the last Italian Hour which we can mark by help of that Parallel, that Line *b III* shall meet with some Astronomical Hour in some Point as *m*, if by the Practice of the Fifth Chapter of this Part we find the Points *l n o* of the Parallel which passes by *m*, and if they be on the Hour before or after that, on which is the Point *m*; we continue to draw the Lines of the Italian or Babylonian Hours by the Points of the Hours of the Parallel *m n o*, and by the Points of the Hours of the Equator, in following the same order as before, and if the Equator be wanting in using the Practice taught at the end of the 5th Chapter, we shall find the Points of another Parallel by the Parallel that is given,

given; and then we may joyn the Points of the Hours on the two Parallels in following the former Order.

There is nothing in all this that deserves any long Explication, nor any other Demonstration, than that we have given in the Two fore-going Chapters.

CHAP. IX.

*Four Astronomique Hours being given, following one another in order, with the Equinoctial Line.
To find the other Hours.*

Fig. 30. **L** Et the four Hour Lines following one another be Aa, Bb, Cc, Dd , with the Equinoctial Line EF ; from a Point a , taken at pleasure in one of the last Lines Aa , having drawn aD which cuts Bb in B , and Cc in g ; also by the same Point a , having drawn aC which cuts Bb in b , let Ab be drawn, which meets Cc in c , and Bg which meets Dd in d : let cb , cd be drawn prolonged to the Equinoctial Line to the Points E, F , the Hour Lines Ee, Ff , drawn by the Points E, F , shall be the Hour Lines

Lines required ; whereof $E e$ shall be distant from $A a$ one Hour , and $F f$ shall be two Hours from $D d$: Therefore $B D$ being prolonged to f in the Line $F f$, and $F d$ to l , in the Line $B b$, having drawn $C f$ which cuts $D d$ in i , $l i$ prolonged shall meet the Equinoctial in M , by which the Hour Line $M m$ shall be between the two Hour Lines $D d$ and $F f$, and these Seven Hour Lines being found, we may have all the rest by the Practice of the *Third Chapter*.

Fig. 31. There are many Cases where Three Hour Lines are sufficient with the Equinoctial and Horizon ; for *Example*, If we have three Hour Lines $a 2$, $b 3$, and $c 4$, and the Equinoctial Line 24 , and Horizontal Line $a c$, having drawn $a 4$, which cuts $b 3$ in d , and having drawn $2 d$ which cuts $c 4$ in f , draw $2 c$, which cuts $b 3$ in e , and $4 e$, which cuts $a 2$ in b ; there ought a strait Line to pass by the three Points $f b b$, which shall meet the Equinoctial in the Point g , which is one Point of the Hour as far from the Hour $b 3$, as is the Hour Line of Six: Therefore if the Hour Line $b e$ be the fourth Hour, $a b$ shall be the third, and $g i$ shall be the second ; but in this *Example*, $b e$ being the third Hour, $g i$ shall be the 12th Hour.

The first Hour between 12 and 2 is found by drawing $g c$, which cuts $a 2$ in k , and $4 k$ which cuts the Hour Line $g i$, which was drawn by the Point g to the Point i , and in drawing $2 i$, which

which cuts gb in n , the Hour Line by the Point n shall be the first Hour.

Demonstration.

By that which hath been demonstrated in the 5th Chapter it is manifest that the Points a and c are in the same Parallel; also that the Points b and d are also in the same; therefore the great Circles that are represented by the Lines cbE , cdF , are equally inclined to the Equator, the one on one side, and the other on the other, but cbF has the same Inclination as abd , because of the equal parts of their Parallels ac , bd ; and there is a space of three Hours between cC and F , and between cC and E ; therefore the Hour which passes by the Point E is the next to that which passes by the Point A , but there are two between the Points D and F .

By the same Construction the Points l and f are in the same Parallel, but fic and liM represent great Circles equally inclined to the Equator, because of the Points l and f which are in the same Parallel, and equally distant from the Point i ; therefore DC and DM represent equal parts of the Equator, which are each of them an Hours distance from one another.

As concerning that which is of three Hour Lines, seeing that by Construction the Points a and f are in the same Parallel, and likewise h and c , the Lines fh and ac which is the Horizon, represent Circles equally inclined to the Equator; therefore there

is as much distance between the Point of the Hour of Six upon the Equinoctial and the Hour Line be, as there is between the same Hour Line be and the Point g. We demonstrate as before that the Point n divides the interval between the Hour Lines gi and a k.

CHAP. X.

A Dial being given which is already drawn, to find the foot of the Style which did serve to draw it, and to determine the height thereof.

Fig. 32. **T**HE Line A B is the Equinoctial Line, and the Distance AB on that Line is the Interval of any six Hours; having divided A B into two equal parts in the Point G, from the Point G as a Center; on the Diameter A B describe the Circle A, S, B, *d*, *f*, and mark the Points *d* and *f* which divide the Semicircle into three equal parts; A F, F D and D B are each the Interval of two Hours on the Equinoctial Line: the Lines *d* D, *f* F ought to meet the Circumference of the Circle at the Point S, and the Line S E P drawn perpendicular to the Equinoctial Line shall be the Substyle.

If we have C the Center of the *Dial*, on the Diameter CE, having described the Semi-circle CZE, and having applied in it the Line EZ equal to ES: ZP being drawn perpendicular to the Substylar Line EP, and meeting it at the Point P, that Point shall be the Foot of the Style, whereof PZ shall be the height.

But if we have not the Center of the *Dial*, having drawn ae parallel to the Equinoctial, and from the Point a having drawn af parallel to AS which meets the Substylar Line in f , from the Point E as a Center, and Semi-diameter ES, describe the Arch z , and from the Point e as a Center and Semi-diameter ef describe the Arch x , and draw the strait Line xz which shall touch the two Arches x and z , and that Line xz shall determine the Inclination of the *Axis* to the Substylar Line, and having drawn Ez perpendicular to xz from the Point E, and from the Point z the strait Line zP perpendicular to the Substylar Line EP, the Point P shall be the foot of the Style, whereof PZ shall be the height.

Demonstration.

If we suppose the Circle ASBd to be on the Plane of the Equinoctial, it is manifest that the Point of the Style ought to be one of the Points of the Circumference of the Semicircle ASB, because

cause the six Intervals of Hours that are on the Equinoctial from A to B, for the Angle ASB, in what place soever the Point S is, ought to be a Right Angle: And also the Lines drawn from that Point S to the Points of the Hours of the Equinoctial Line, ought to comprehend equal Angles each of 15 degrees in the same Point S; therefore the Arches AF, fd, and dB being each of 60 degrees, shall subtend Angles of 30 degrees, which are each in value two Hours, and consequently the Lines fF and dD shall be Hour Lines on the Equinoctial; therefore they meet in the same Point S upon the Circumference of the Circle which determines the Point of the Style in respect of the Equinoctial, and the Line SE drawn perpendicular to the Equinoctial Line, doth there give the Point E which is one of the Points of the Substylar Line; but the Substylar Line ought to be perpendicular to the Equinoctial Line; therefore the same strait Line SEP shall be the Substylar.

The Axis ought to make a Right Angle with the Plane of the Equinoctial, therefore the Line ES which is equal to that which ought to meet the Axis on the Plane of the Equinoctial, ought to make a Right Angle with the Axis.

Therefore if we have c the Center of the Dial, and if we describe the Semi-circle CZE, applying EZ equal to ES in that Semi-circle, it is manifest that CZE shall be a Right Angle, and that the Line CZ represents the Inclination of the Axis to the Substylar Line CE, and that the Point Z represents the

the Point of the Style which has served to draw the Dial: Therefore ZP which is drawn perpendicular to CE , shall be the height of the Style, whereof the Point P shall be the foot.

But if we have not the Center of the Dial, it is manifest by Construction that the two Lines ES & es having one of their extrems in the Points E & e , the other will be had in the Axis; if the Line that joyns the two last extrems be perpendicular to those two Lines which ought to be parallel; but it is also evident that the Line xz which touches the two Arches of Circles which have for their Semi diameters ES , es , shall be perpendicular to the two strait Lines Ez , ex , which come from the Center to the touching Points; therefore the Line xz gives the Inclination of the Axis to the Substyle, and the Line zP drawn perpendicular to the Substyle, shall determine the height of the Style, whereof P shall be the foot.

CHAP.

CHAP. XI.

To place the Axis.

IF we would have the Hours shewn only by the shadow of the Point of the Style, we ought to make and to place the Style after such a manner as may serve without changing of it.

We may give it divers forms, but one of the best is, to make it waved to the end, that the Shadow thereof may not unite with the Hour Lines in any place, and that we may always know that it is but only the Shadow of the Point that serves to shew the Hours.

But if you would have a portion of the *Axis* to shew the Hours, and that the *Axis* be represented by an Iron Rod, the Style we have placed ought to have the Point very small, that it may enter into a little hole made in the Rod, so as the Point of the Style may exactly answer to the middle of the thickness of the Rod; the Style may remain, if you would have it, to support the *Axis*; but if the *Axis* be not very long, and if it be strong enough to sustain it self alone being fastened

fastened at one end, we may take away the *Style* when the *Axis* is fixed on the Surface of the *Dial*. We may do the same, if we fasten to the end of the *Style* a Point of an *Iron Wyer*, which there may be very small, and may take away but the half of the thickness of the Rod, so as the *Dial* being drawn to that *Point*, there is nothing to be done but to take it away to place the *Axis*; whereof the middle of the thickness ought to answer to that *Point*; therefore whether the *Style* remains to uphold the *Axis*, or whether we take it away when the *Axis* is fix'd in its place, we must fasten it to the end of the *Style* to stay the end thereof, which ought to answer to the Center of the *Dial* if it have any.

Fig. 33. We may make the *Rod* which serves for the *Axis* as it is marked in the Figure, so as the hole signified by A may be made to lodge the Point of the *Style*, and that it be let in as far as the middle of the thickness of the Rod, the Point B which answers also to the middle of the *Rod*, ought to be applied exactly to the Center of the *Dial*, this Rod being so stayed at the Point B and at the Point A, we must fasten the Foot C on the *Plane* of the *Dial*.

But if you would not have a Foot to the *Axis* as G, and that you would only fix the Rod to the Center of the *Dial*, you must draw divers Lines which may pass by the Center of

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the

the *Dial*, and stay the Rod on the Point of the *Style* A, and by any other place, so as the end may enter in a hole made in the *Plane* of the *Dial* at the place of the Center, may be divided by the middle of its thickness by each Line that passes by the Center.

Fig. 34. Also we may make use of a thin Plate which must be cut according to the Inclination of the *Axis* with the Substylar Line; it must be set perpendicularly on the *Plane* of the *Dial*, in applying one of its sides to the Substylar Line, and the other side passing by the Point of the *Style* shall serve for the *Axis*. If you would draw on the *Plate* a perpendicular Line to that side of it which is applied to the Substylar Line, and equal to the height of the *Style*, the which shall represent the *Style*, so as the end of that Line meet with the other side of the *Plate*, that Line when the Plate shall be set in its place, shall answer to the *Style*. See here divers Figures of these sorts of *Plates* with their feet to fasten them on the *Plane* of the *Dial*.

CHAP. XII.

To Draw Dials by Reflection.

TO make a *Dial* that may shew the Hours by the Reflection of the Light of the *Sun*, you must make use of a small piece of polished Metal very even and flat; of a round form, and of about an Eighth part of an Inch in Diameter; and having placed it, and fastened it in a place very stable and immovable, we mark the Points of Light on the *Plane* where we intend to draw the *Dial*, which serve instead of the *Points* of Shadow, the middle of the Mirror or Glass ought to be considered as the Point of a *Style*, whereof we find the foot in drawing from the middle of the Glass a Line perpendicular to the Plane of the Dial, the Point where this Line meets with the Plane of the Dial, shall be the Foot of the *Style*.

We may find the Substylar Line, the Equinoctial Line, the Center of the Dial and Meridian by the practices, where we make no use of the Horizontal Line, nor of the height of the Pole.

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Having

Having found the Equinoctial Line, and the Point where the Meridian Line intersects it, we draw the Hours, following the Methods of the Second Part of this Treatise.

Here it is to be observed that if the Inclination of the Glass be never so little changed, all the *Dial* will be considerably changed. Therefore these sorts of *Dials* do very hardly remain many years in a good condition; for there always happens some alterations to the Wall on which they are fixt.

But if in place of the Glass we fill some small Vessel either of Glass or Potters Earth, of about an Inch in Diameter, with Water or Quick Silver, that Vessel being put upon a place marked on some Transum of a Window or the like, so as you may always set it in the same place again, if you would take it away, the Reflection of the Light from the Water or Quick Silver, shall give the Hours on the *Dial* of which we must draw the Lines, as has been taught in the First and Second Part of this Work, in observing only that the middle of the *Superficies* of the Water or Quick Silver, serves for the Point of the *Style*, and that the Operations which are made on the Horizon below for the *Dials*, which gives the Hour by the Shadow of a Point, ought to be made on the Horizon above, and that which is made above in those *Dials*, in these *Dials* to be made below.

CHAP.

CHAP. XIII.

Concerning the Table of the Suns Declination, and of those of the Difference of Meridians of divers considerable places in respect of Paris.

THE *Tables* of the Declination of the *Sun* which are at the end of this Work, is calculated for the Meridian of *Paris*, and for each day at Noon, on the side are the Differences between the Declination of one day and that of the next day following; they are made for Four Years following one another. The First Year begins in 1681, which is the First after the *Bissextile* or *Leap Year*, the Second is the following Year 1682, the Third is 1683, and the Fourth Year 1684, is *Bissextile* or *Leap Year*.

Then afterwards there follows a *Table* of the Differences of Meridians of the principal places of the Earth in respect of *Paris*; it is calculated in Hours and Minutes, and it serves to find the Declination of the *Sun* in all those places at any Hour proposed. There is also another *Table* which is joyned to that, in which we may find

the Latitude or Height of the Pole of the same places.

We begin the Days in these Tables and in the following Calculations, from each Day at Noon, and continue it to the next following Day at Noon, and we count the Hours to 24, soas the Third Hour in the Morning of any day proposed, is the Fifteenth Hour after Noon of the preceding day.

R U L E I.

To continue the Tables of the Suns Declination.

IF you would have the Declination of the Sun for the Years following those that are calculated, you must add to the Declination increasing of each day, and subtract from the Declination decreasing after Four Years fully past, one Minute for 32 Minutes of Difference, half a Minute for 16 Minutes of Difference, a quarter of a Minute for 8 Minutes of Difference, and so of the rest in proportion.

Exam-

Example.

If you would know the Declination of the *Sun* for the 5th of *January* at Noon for the Year 1685, which is the First Year after the *Bissex-tile* or *Leap Year*, and is the Fourth that follows to the Year 1681, whereof we have the Calculation in the Table. I find that the Difference between the 5th and 6th of *January* 1681, is 12 Minutes, and the Declination is decreasing; therefore I see that there must be subtracted about $\frac{2}{3}$ of a Minute from the Declination of the 5th of *January* 1681, the which is 20 Degrees 58 Minutes; and we shall have 20 degrees 57 Minutes $\frac{2}{3}$ for the Declination of the *Sun* the 5th of *January* at Noon in the Year 1685; if there be Eight Years passed between the Year in which you would have the *Sun*s Declination, and that which answers to it in this Table, that is to say, that which is equally distant from the *Leap Year*, you must double this Correction; if it be 12 Years, you must triple the Correction, and so forth.

H 4

RULE

R U L E II.

To find the Declination of the Sun at all Hours of the Day.

HAVING found in the Table of differences of the *Suns* Declination, the difference of the Declination of the *Sun* between the given Day and the next following Day, take the part proportional of that Difference answerable to the given Hours, which we add to the Declination of the same Day if it increase, but subtract it if it decrease.

Example.

As to know the Declination of the *Sun* on the 15th day of *March* at 4 a Clock in the Afternoon for the Year 1683, I find in the Table that the Difference between the 15th and 16th day is 23 minutes and $\frac{1}{2}$ (for there is sometimes 24 minutes and sometimes 23 minutes) and the Declination of the *Sun* for the 15th of *March* at Noon in the Year proposed 1683, being 1 degree 59 minutes, to which here must be added the proportional

tional part of the Difference for 4 Hours, which is the sixth part of 24 Hours; therefore you must take the sixth part of 23 minutes and $\frac{1}{2}$, which is about 4 minutes, which being added to the Declination found 1 degree 59 minutes, because the Declination increases, and we shall have 2 degrees 3 minutes, the Declination increasing on that Day at the required Hour; but if the Declination were decreasing, we must subtract the part proportional of the Declination found in the Table.

This ought to be understood of the Declinations of the *Sun* for the Meridian of *Paris*, as we find them in this Table, but for other places on the Earth, they are to be reduced according to the following *Rule*.

RULE III.

To know the Declination of the Sun at a given Hour in any place that is set down in the Table.

THE Table of Differences of Meridians of places which are here set down in respect of *Paris*, with the word *add* or *subtract* shews how

how much later or sooner it is Noon at *Paris* than at that place; the chief use whereof is to know the Declination of the *Sun* at a given Hour in any place set down in the Table.

Therefore if you would know the Declination of the *Sun* at a given Hour in any place that is in the Table, you must find the Difference of its Meridian in respect of *Paris*, and joyn it to the given Hour, if there be *add* after it, and if the Sum exceed 24 Hours, we take the over-plus in the next following day to that as was given; but if there be found *subtract*, we take that Difference of Meridians from the Hour proposed; but if the Hour be too little, add 24 Hours to it, and then subtract, and the remainder shall be attributed to the fore-going Day.

Having therefore found this Sum or Difference of Hours, we find the Declination of the *Sun* for that Hour at *Paris* by the Second Rule, and you shall have that which you require for the given place.

Example.

If we would know for *Rome* the Declination of the *Sun* for the 22th day of *August* 1682, at 4 a Clock in the Morning.

First, Because the Hour proposed is in the Morning, I reduce it to 16 Hours after Noon on the 21th day of *August*, and having found in the

the Table of Differences of Meridians, that for *Rome* I must substract 47 minutes, therefore I take 47 minutes from 16 hours, and there remains 15 hours and 13 minutes after Mid-day on the 21th of *August* 1682; and by the Second Rule I find that the Declination for that day and that Hour is at *Paris* 8 degrees and 15 minutes, which is that which was required for 4 a Clock in the Morning the 22th day of *August* 1682 at *Rome*.

Another Example.

If you would know for *Peking* in *China* the Declination of the Sun at 2 a Clock in the Morning on the 25th of *July* 1684.

First, because the Hour proposed is before Noon, I reduce it to the fore-going day, which will be the 24th of *July* at 14 Hours after Noon, and I find in the Table of Differences of Meridians, that for *Peking* I must substract 7 hours 45 minutes, the which being substracted from 14 hours, there remains 6 hours 15 minutes; therefore I search by the Second Rule the Declination of the Sun for *Paris* on the 24th of *July*, at 6 a Clock 15 minutes after Noon, in the Year 1684, and I find 17 degrees, and about 13 minutes which is the Declination of the Sun for *Peking* on the day and hour proposed.

Another Example.

I would know at *Quebec* the Declination of the *Sun* on the 25th of *March* 1683, at 10 a Clock in the Morning, which being reduced, will be 22 Hours after Noon on the 24th of *March*, and I find in the Table that I must add for *Quebec* 4 Hours 36 minutes, therefore we have the 24th of *March* at 2 a Clock 36 minutes after Noon. For which time in the Year 1683, I find the *Sun's* Declination to be 5 degrees and about 30 minutes, which is that for *Quebec* on the day and hour required..

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AN ADVERTISEMENT concerning the Figures.

Here you must observe that the *Dials* drawn in these Figures, are not made expressly for any place; for it is impossible to make the Magnitude of the Lines which we must use equal to those that are here drawn: we ought only to follow the Precepts, and not to measure with the Compasses the length of the Lines, to see if they agree: For *Example*, altho we say make the Line *as* equal to *AS*, yet these two Lines are not equal in the Figure, because that sometimes one of the ends of the Line *AS* as *S* being the Point of the Style which is not in the *Plane*, the apparent Magnitude of that Line is not the true Magnitude, and it also happens often-times that the Lines of the Figure answers not among themselves according to the Discourse which was done to order the place of the Figures: it is sufficient to observe well after what Manner, of what
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Magnitude, and in what Angles we prescribe
to draw the Lines without depending on the
Figure which serves but to help the Imagina-
tion, and to guide you in the Operations, seeing
that it is almost impossible to meet with two
like *Dials* among a great number which we make
on the *Planes* proposed, as it is found ordinarily.

ADVANTAGES
CONCERNING THE FIGURES

For you must observe that the Lines drawn
in these Figures are not really equal in
place; for it is impossible to make the Mag-
nitude of the Lines which we must use equal to
those that are here drawn: we ought only to fol-
low the Precepts, and not to measure with the
compasses the length of the Lines, to see if
they agree: for Example, when we say make
a Line *a* equal to *A B*, yet these two Lines
are not equal in the Figure, because the long-
est one of the ends of the Line *A B* is being
the Point of the Style which is not in the Figure.
The apparent Magnitude of that Line is double
the Magnitude, and it also happens often times
that the Lines of the Figure answers not among
themselves according to the Distance which was
given to order the place of the Figure: it is thus
that to observe well after what Manner of what
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A

TABLE

Of the Suns Declination

For the Year 1681,

Which was the First after

Bissextile or Leap-year.

For the year 1681.

Days	January		Diff. M.	February.		Diff. M.
	D.	M. South.		D.	M. South.	
1	21	41		13	42	
2	21	31	10	13	22	20
3	21	20	11	13	2	20
4	21	9	11	12	42	20
5	20	58	11	12	21	21
6	20	46	11	12	0	21
7	20	34	12	11	39	21
8	20	22	12	11	18	21
9	20	9	12	10	57	21
10	19	56	13	10	35	22
11	19	42	14	10	13	22
12	19	28	14	9	51	22
13	19	14	14	9	29	22
14	18	59	15	9	7	22
15	18	44	15	8	45	22
16	18	29	15	8	22	23
17	18	13	16	7	59	23
18	17	57	16	7	36	23
19	17	41	17	7	13	23
20	17	24	17	6	50	23
21	17	7	17	6	27	23
22	16	50	18	6	4	23
23	16	32	18	5	41	23
24	16	14	18	5	18	24
25	15	56	18	4	54	23
26	15	38	19	4	31	23
27	15	19	19	4	8	24
28	15	0	19	3	44	24
29	14	41	19			
30	14	22	20			
31	14	2				

Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
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For the Year 1682

Diff. M.	Days	March		Diff. M.	Days	April		Diff. N.
		D.	M. No.			D.	M. North	
20	1	31	20	23	1	8	39	22
20	2	2	57	24	2	9	1	22
20	3	2	33	23	3	9	23	21
21	4	2	10	23	4	9	44	21
21	5	1	47	24	5	10	5	21
21	6	1	23	24	6	10	26	21
21	7	0	59	24	7	10	47	21
21	8	0	35	24	8	11	8	21
22	9	0	11	23	9	11	29	21
22	10	0	12	24	10	11	50	20
22	11	0	36	24	11	12	10	20
22	12	1	0	23	12	12	30	20
22	13	1	23	24	13	12	50	20
22	14	1	47	24	14	13	10	19
23	15	2	11	23	15	13	29	19
23	16	2	34	23	16	13	48	19
23	17	2	57	24	17	14	7	19
23	18	3	21	23	18	14	26	19
23	19	3	44	23	19	14	45	18
23	20	4	7	24	20	15	3	18
23	21	4	31	23	21	15	21	18
23	22	4	54	23	22	15	39	17
23	23	5	17	23	23	15	36	17
24	24	5	40	23	24	16	13	17
23	25	6	3	23	25	16	30	17
23	26	6	26	22	26	16	47	17
23	27	6	48	22	27	17	4	16
24	28	7	10	22	28	17	20	16
24	29	7	32	23	29	17	36	16
	30	7	55	22	30	17	52	15
	31	8	17	22				

For the Year 1681.

Days	May		Diff. N.
	D.	M.	
	North		
1	18	7	15
2	18	22	14
3	18	36	15
4	18	51	14
5	19	5	14
6	19	19	13
7	19	32	13
8	19	45	13
9	19	58	12
10	20	10	12
11	20	22	12
12	20	34	11
13	20	45	11
14	20	56	11
15	21	7	10
16	21	17	10
17	21	27	10
18	21	37	9
19	21	46	9
20	21	55	9
21	22	4	8
22	22	12	8
23	22	20	7
24	22	27	7
25	22	34	6
26	22	40	6
27	22	46	6
28	22	52	5
29	22	57	5
30	23	2	5
31	23	7	5

Days	June		Diff. N.
	D.	M.	
	North		
1	23	11	4
2	23	15	4
3	23	18	4
4	23	21	4
5	23	23	4
6	23	25	4
7	23	27	4
8	23	28	4
9	23	29	4
10	23	29	4
11	23	29	4
12	23	29	4
13	23	28	4
14	23	27	4
15	23	25	4
16	23	23	4
17	23	21	3
18	23	18	3
19	23	15	3
20	23	11	4
21	23	7	5
22	23	2	5
23	22	57	5
24	22	52	6
25	22	46	6
26	22	40	6
27	22	34	7
28	22	27	7
29	22	20	8
30	22	12	8

For the Year 1681.

Diff. M.	July			Diff. M.	August			Diff. M.
	D.	M.	North.		D.	M.	North.	
1	22	4	8	1	15	6	18	
2	21	56	9	2	14	48	18	
3	21	47	9	3	14	29	18	
4	21	38	10	4	14	11	19	
5	21	28	10	5	13	52	19	
6	21	18	10	6	13	33	19	
7	21	8	10	7	13	14	20	
8	20	57	11	8	12	54	20	
9	20	46	11	9	12	34	20	
10	20	35	11	10	12	14	20	
11	20	23	12	11	11	54	20	
12	20	11	12	12	11	34	20	
13	19	59	12	13	11	14	21	
14	19	46	13	14	10	53	21	
15	19	33	13	15	10	32	21	
16	19	20	13	16	10	11	21	
17	19	6	14	17	9	50	21	
18	18	52	14	18	9	29	21	
19	18	38	14	19	9	8	22	
20	18	23	15	20	8	46	22	
21	18	8	15	21	8	24	22	
22	17	53	15	22	8	2	22	
23	17	38	15	23	7	40	22	
24	17	22	16	24	7	18	22	
25	17	6	16	25	6	56	22	
26	16	50	17	26	6	34	23	
27	16	33	17	27	6	11	23	
28	16	16	17	28	6	48	23	
29	15	59	17	29	5	25	23	
30	15	42	18	30	5	2	23	
31	15	24		31	4	39	23	

For the Year 1681.

Days	September		Diff. M.
	No.	M.	
1	4	16	23
2	3	55	23
3	3	30	23
4	3	7	23
5	2	44	23
6	2	21	23
7	1	58	23
8	1	34	24
9	1	11	23
10	0	47	24
11	0	24	23
12	0 South	0	24
13	0	23	23
14	0	47	24
15	1	10	23
16	1	34	24
17	1	57	23
18	2	21	24
19	2	44	23
20	3	7	23
21	3	31	24
22	3	54	23
23	4	17	23
24	4	41	24
25	5	4	23
26	5	27	23
27	5	50	23
28	6	13	23
29	6	36	23
30	6	59	23
31	6	4	23

Days	October		Diff. M.
	D.	M.	
1	7	22	27
2	7	44	23
3	8	7	22
4	8	29	22
5	8	51	22
6	9	13	22
7	9	35	22
8	9	57	22
9	10	19	22
10	10	41	21
11	11	2	21
12	11	23	21
13	11	44	21
14	12	5	21
15	12	26	21
16	12	47	20
17	13	7	20
18	13	27	20
19	13	47	20
20	14	7	19
21	14	26	19
22	14	45	19
23	15	4	19
24	15	23	19
25	15	42	18
26	16	0	18
27	16	18	18
28	16	36	17
29	16	53	17
30	17	10	17
31	17	27	16

For the Year 1681.

Diff. M.	Days	November		Diff. M.		Days	December		Diff. M.
		D.	M.				M.	D.	
		South					South		
22	1	17	43	16		1	23	8	4
23	2	17	59	16		2	23	12	4
22	3	18	15	16		3	23	16	4
22	4	18	31	15		4	23	19	3
22	5	18	46	15		5	23	22	3
22	6	19	1	15		6	23	24	2
22	7	19	16	15		7	23	26	2
22	8	19	30	14		8	23	28	2
22	9	19	44	14		9	23	29	1
21	10	19	57	13		10	23	29	0
21	11	20	10	13		11	23	29	0
21	12	20	23	12		12	23	29	0
21	13	20	35	12		13	23	28	1
21	14	20	47	12		14	23	27	1
21	15	20	59	11		15	23	25	2
20	16	21	10	11		16	23	23	2
20	17	21	21	10		17	23	20	3
20	18	21	31	10		18	23	17	3
20	19	21	41	10		19	23	13	4
19	20	21	51	9		20	23	9	4
19	21	22	0	9		21	23	4	5
19	22	22	9	8		22	22	59	5
19	23	22	17	8		23	22	53	6
19	24	22	25	7		24	22	47	6
18	25	22	32	7		25	22	41	7
18	26	22	39	7		26	22	34	7
18	27	22	46	6		27	22	27	8
17	28	22	52	6		28	22	19	8
17	29	22	58	5		29	22	11	9
17	30	23	3	5		30	22	2	9
16				5		31	21	53	10

For the Year 1681.

A

TABLE

Of the Suns Declination

For the Year 1682.

Being the Second after

Bisextile or Leap-Year.

For the Year 1682.

January			February		
Days	D.	M.	Days	D.	M.
	South			South	
1	21	43	1	13	47
2	21	33	2	13	27
3	21	23	4	15	37
4	21	12	4	12	46
5	21	1	5	12	26
6	20	49	6	12	15
7	20	37	7	11	44
8	20	25	8	11	23
9	20	12	9	11	1
10	19	59	10	10	40
11	19	45	11	10	18
12	19	31	12	9	56
13	19	17	13	9	34
14	19	3	14	9	12
15	18	48	15	8	50
16	18	33	16	8	27
17	18	17	17	8	5
18	18	1	18	7	42
19	17	45	19	7	19
20	17	28	20	6	16
21	17	11	21	6	33
22	16	54	22	6	10
23	16	37	23	5	47
24	16	19	24	5	24
25	16	1	25	5	10
26	15	43	26	4	37
27	15	24	27	4	13
28	15	5	28	3	50
29	14	46			
30	14	27			
31	14	7			

For the Year 1682.

Days	March		Diff. M.
	D.	M.	
	So.	No.	
1	3	26	23
2	3	3	24
3	2	39	24
4	2	15	23
5	1	52	24
6	1	28	24
7	1	4	23
8	10	41	24
9	0	17	24
10	0	No. 7	23
11	0	30	24
12	0	54	24
13	1	18	23
14	1	41	24
15	2	5	23
16	2	28	24
17	2	52	23
18	3	15	23
19	3	38	24
20	4	2	23
21	4	25	23
22	4	48	23
23	5	11	23
24	5	34	23
25	5	57	22
26	6	20	22
27	6	42	22
28	7	5	22
29	7	27	22
30	7	49	22
31	8	11	22

Days	April		Diff. M.
	D.	M.	
	North		
1	8	33	22
2	8	55	22
3	9	17	22
4	9	39	21
5	10	0	21
6	10	21	21
7	10	42	21
8	11	3	21
9	11	24	21
10	11	45	20
11	12	5	20
12	12	25	20
13	12	45	20
14	13	5	19
15	13	24	19
16	13	43	19
17	14	2	19
18	14	21	19
19	14	40	18
20	14	58	18
21	15	16	18
22	15	34	18
23	15	52	17
24	16	9	17
25	16	26	17
26	16	43	17
27	17	0	16
28	17	16	16
29	17	32	16
30	17	48	15

1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
11	11
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
20	20
21	21
22	22
23	23
24	24
25	25
26	26
27	27
28	28
29	29
30	30
31	31

For the Year 1682

Diff. M.

22
22
22
21
21
21
21
21
21
20
20
20
20
19
19
19
19
18
18
18
17
17
17
16
16
16
15

Days	May		Diff. M.
	D.	M. North.	
1	18	3	15
2	18	18	15
3	18	33	14
4	18	47	14
5	19	1	14
6	19	15	14
7	19	29	13
8	19	42	13
9	19	55	12
10	20	7	12
11	20	19	12
12	20	31	12
13	20	43	11
14	20	54	11
15	21	5	10
16	21	15	10
17	21	25	9
18	21	35	9
19	21	44	9
20	21	53	8
21	21	2	8
22	22	10	7
23	22	18	7
24	22	25	7
25	22	32	6
26	22	39	6
27	22	45	6
28	22	51	5
29	22	56	5
30	23	1	5
31	23	6	5

Days	June		Diff. M.
	D.	M. North.	
1	23	10	4
2	23	14	4
3	23	17	4
4	23	20	4
5	23	23	4
6	23	25	4
7	23	27	4
8	23	28	4
9	23	29	4
10	23	29	4
11	23	29	4
12	23	29	4
13	23	28	4
14	23	27	4
15	23	26	4
16	23	24	4
17	23	22	4
18	23	19	4
19	23	16	4
20	23	13	4
21	23	10	4
22	23	7	4
23	22	59	3
24	22	54	3
25	22	48	3
26	22	42	3
27	22	36	3
28	22	29	3
29	22	22	3
30	22	14	3
31	22	7	3

For the Year 1682.

Days	July		Diff. N.
	D.	M. North.	
1	22	6	
2	21	58	8
3	21	49	9
4	21	40	9
5	21	30	10
6	21	20	10
7	21	10	10
8	21	0	10
9	20	49	11
10	20	38	11
11	20	26	12
12	20	14	12
13	20	2	12
14	19	49	13
15	19	36	13
16	19	23	13
17	19	9	14
18	18	55	14
19	18	41	14
20	18	27	14
21	18	12	15
22	17	57	15
23	17	41	16
24	17	25	16
25	17	9	16
26	16	53	16
27	16	37	16
28	16	20	17
29	16	3	17
30	15	46	17
31	15	28	18

Days	August		Diff. N.
	D.	M. North.	
1	15	16	
2	14	52	18
3	14	34	18
4	14	15	19
5	13	56	19
6	13	37	19
7	13	18	19
8	12	59	20
9	12	39	20
10	12	19	20
11	11	59	20
12	11	39	20
13	11	19	21
14	10	58	21
15	10	37	21
16	10	16	21
17	9	55	21
18	9	34	21
19	9	13	22
20	8	51	22
21	8	29	22
22	8	7	22
23	7	45	22
24	7	23	22
25	7	1	23
26	6	38	23
27	6	16	22
28	5	54	22
29	5	31	23
30	5	8	23
31	4	45	23

For the Year 1682.

Days	September		Diff. N.	Days	October		Diff. N.
	D.	M.			D.	M.	
	No.	So			South		
1	4	22	23	1	7	16	23
2	3	59	23	2	7	39	22
3	3	36	23	3	8	1	23
4	3	13	23	4	8	24	22
5	2	50	24	5	8	46	22
6	2	26	23	6	9	8	22
7	2	3	23	7	9	30	22
8	1	40	24	8	9	52	22
9	1	16	23	9	10	14	22
10	0	53	23	10	10	36	21
11	0	30	24	11	10	57	21
12	South	66	23	12	11	18	21
13	0	17	24	13	11	39	21
14	0	41	23	14	12	0	21
15	1	4	24	15	12	21	21
16	1	28	24	16	12	42	20
17	1	52	23	17	13	2	20
18	2	15	23	18	13	22	20
19	2	38	23	19	13	42	20
20	3	2	23	20	14	2	20
21	3	25	23	21	14	22	19
22	3	48	24	22	14	41	19
23	4	12	23	23	15	0	19
24	4	35	23	24	15	19	18
25	4	58	24	25	15	37	18
26	5	22	23	26	15	55	18
27	5	45	23	27	16	13	18
28	6	8	23	28	16	31	18
29	6	31	22	29	16	49	17
30	6	53	23	30	17	6	17
				31	17	23	

For the Year 1682.

Days	November		Diff. N.		Days	December		Diff. N.
	D.	M.				M.	D.	
	South					South		
1	17	40	16		1	23	7	4
2	17	36	16		2	23	11	4
3	18	12	15		3	23	15	4
4	18	27	15		4	23	18	3
5	18	42	15		5	23	21	3
6	18	57	15		6	23	24	2
7	19	12	14		7	23	26	2
8	19	26	14		8	23	28	1
9	19	40	14		9	23	29	1
10	19	54	13		10	23	29	0
11	20	7	13		11	23	29	0
12	20	20	12		12	23	29	0
13	20	32	12		13	23	28	1
14	20	44	12		14	23	27	1
15	20	56	11		15	23	25	2
16	21	7	11		16	23	23	2
17	21	18	11		17	23	20	3
18	21	29	10		18	23	17	3
19	21	39	10		19	23	14	3
20	21	49	9		20	23	10	4
21	21	58	9		21	23	5	5
22	22	7	8		22	23	0	5
23	22	15	8		23	22	55	6
24	22	23	8		24	22	49	7
25	22	31	7		25	22	42	7
26	22	38	7		26	22	35	7
27	22	45	6		27	22	28	8
28	22	51	6		28	22	20	8
29	22	57	5		29	22	12	8
30	23	2	5		30	22	4	9
					31	21	55	

Dff.
M.
4
4
3
3
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TABLE

Of the Suns Declination For the Year 1683.

Being the Third after

Bissextile or Leap-Year.

For the Year 1683.

Days	January		Diff. M		Days	February		Diff. M
	D.	M.				D.	M.	
	South					South		
1	21	46	10		1	13	52	20
2	21	36	10		2	13	32	20
3	21	26	11		4	13	12	20
4	21	15	11		4	12	52	21
5	21	4	12		5	12	31	21
6	20	52	12		6	12	10	21
7	20	40	12		7	11	49	21
8	20	28	12		8	11	28	21
9	20	15	13		9	11	7	22
10	20	2	13		10	10	45	22
11	19	49	13		11	10	23	22
12	19	35	14		12	10	1	22
13	19	21	14		13	9	39	22
14	19	6	15		14	9	17	22
15	18	51	15		15	8	55	22
16	18	36	15		16	8	33	23
17	18	21	15		17	8	10	23
18	18	5	16		18	7	47	23
19	17	49	16		19	7	24	23
20	17	32	17		20	7	1	23
21	17	15	17		21	6	38	23
22	16	58	17		22	6	15	23
23	16	41	17		23	5	52	23
24	16	23	18		24	5	29	23
25	16	5	18		25	5	6	24
26	15	47	18		26	4	42	23
27	15	29	18		27	4	19	23
28	15	10	19		28	3	56	24
29	14	51	19					
30	14	32	19					
31	14	12	20					

For the Year 1683.

Days	March		Diff. M.
	D.	M. No.	
1	3	32	24
2	3	8	23
3	2	45	23
4	2	21	24
5	1	57	24
6	1	34	23
7	1	10	24
8	0	46	23
9	0	23	24
10	0	No. 1	24
11	0	25	24
12	0	49	23
13	1	12	24
14	1	36	23
15	1	59	24
16	2	23	23
17	2	46	24
18	3	10	23
19	3	33	23
20	3	56	23
21	4	19	23
22	4	42	23
23	5	5	23
24	5	28	23
25	5	51	23
26	6	14	22
27	6	37	23
28	6	59	23
29	7	22	22
30	7	44	22
31	8	6	22

Days	April		Diff. M.
	D.	M. North	
1	8	28	22
2	8	50	22
3	9	12	22
4	9	34	21
5	9	55	21
6	10	16	21
7	10	37	21
8	10	58	21
9	11	19	21
10	11	40	21
11	12	0	20
12	12	20	20
13	12	40	20
14	13	0	20
15	13	20	20
16	13	39	19
17	13	58	19
18	14	17	19
19	14	36	18
20	14	54	18
21	15	12	18
22	15	30	18
23	15	48	17
24	16	5	17
25	16	22	17
26	16	39	17
27	16	56	16
28	17	12	16
29	17	28	16
30	17	44	15

For the Year 1683.

Days	May		Diff. M.
	D.	M. North	
1	17	59	15
2	18	14	15
3	18	29	14
4	18	44	14
5	18	58	14
6	19	12	14
7	19	26	13
8	19	39	13
9	19	52	12
10	19	5	12
11	20	17	12
12	20	29	12
13	20	40	11
14	20	51	11
15	20	2	10
16	21	13	10
17	21	23	10
18	21	33	9
19	21	42	9
20	21	51	9
21	22	0	8
22	22	8	8
23	22	16	7
24	22	23	7
25	22	30	7
26	22	37	6
27	22	43	6
28	22	49	6
29	22	55	5
30	23	0	5
31	23	5	4

Days	June		Diff. M.
	D.	M. North	
1	23	9	4
2	23	13	3
3	23	16	3
4	23	19	2
5	23	22	2
6	23	24	2
7	23	26	1
8	23	27	1
9	23	28	1
10	23	29	0
11	23	29	0
12	23	29	0
13	23	28	1
14	23	27	1
15	23	26	1
16	23	24	2
17	23	22	3
18	23	19	3
19	23	16	3
20	23	13	4
21	23	9	4
22	23	5	5
23	23	0	5
24	22	55	6
25	22	49	6
26	22	43	6
27	22	37	7
28	22	30	7
29	22	23	7
30	22	16	8

Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
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For the Year 1683.

July			August		
Days	D.	M.	Days	D.	M.
	North.			North.	
1	22	8	1	15	14
2	22	0	2	14	56
3	21	51	3	14	38
4	21	42	4	14	20
5	21	33	5	14	1
6	21	23	6	13	42
7	21	13	7	13	23
8	21	2	8	13	4
9	21	51	9	12	44
10	20	40	10	12	24
11	20	29	11	12	4
12	20	17	12	11	44
13	20	5	13	11	24
14	19	52	14	11	3
15	19	39	15	10	42
16	19	26	16	10	21
17	19	13	17	10	0
18	18	59	18	9	39
19	18	45	19	9	18
20	18	30	20	8	56
21	18	15	21	8	34
22	18	0	22	8	12
23	17	45	23	7	50
24	17	29	24	7	28
25	17	13	25	7	6
26	16	57	26	6	44
27	16	41	27	6	22
28	16	24	28	5	59
29	16	7	29	5	36
30	15	50	30	5	13
31	15	32	31	4	51

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For the Year 1683.

Days	September		Diff. M.	Days	October		Diff. M.
	D.	M.			D.	M.	
	No.	So.			South		
1	4	28	23	1	7	11	22
2	4	5	23	2	7	33	23
3	3	42	23	3	7	56	22
4	3	19	24	4	8	18	23
5	2	55	23	5	8	41	22
6	2	32	23	6	9	3	22
7	2	9	24	7	9	25	22
8	1	45	23	8	9	47	22
9	1	22	23	9	10	9	21
10	0	59	24	10	10	30	22
11	0	35	23	11	10	52	21
12	South	12	24	12	11	13	21
13	0	12	23	13	11	34	21
14	0	35	24	14	11	55	21
15	0	59	23	15	12	16	21
16	1	22	24	16	12	37	20
17	1	46	23	17	12	57	20
18	2	9	24	18	13	17	20
19	2	33	23	19	13	37	20
20	2	56	23	20	13	57	20
21	3	19	24	21	14	7	19
22	3	43	23	22	14	36	19
23	4	6	23	23	14	55	19
24	4	29	24	24	15	14	19
25	4	53	23	25	15	33	18
26	5	16	23	26	15	51	18
27	5	39	23	27	16	9	18
28	6	2	23	28	16	27	17
29	6	25	23	29	16	44	17
30	6	48	23	30	17	1	17
				31	17	18	17

For the Year 1683.

Diff. M.	Days	November		Diff. M.	Days	December		Diff. M.
		D.	M. South			M.	D. South	
22	1	17	35	16	1	23	6	4
23	2	17	51	16	2	23	10	4
22	3	18	7	16	3	23	14	4
23	4	18	23	16	4	23	18	4
22	5	18	39	15	5	23	21	3
22	6	18	54	15	6	23	23	2
22	7	19	9	14	7	23	25	2
22	8	19	23	14	8	23	27	1
21	9	19	37	14	9	23	28	1
22	10	19	51	13	10	23	29	0
21	11	20	4	13	11	23	29	0
21	12	20	17	13	12	23	29	1
21	13	20	30	12	13	23	28	1
21	14	20	42	12	14	23	27	1
21	15	20	54	11	15	23	26	2
20	16	21	5	11	16	23	24	3
20	17	21	16	11	17	23	21	3
20	18	21	27	10	18	23	18	3
20	19	21	37	10	19	23	15	4
20	20	21	47	9	20	23	11	5
19	21	21	56	9	21	23	6	5
19	22	22	5	8	22	23	1	5
19	23	22	13	8	23	22	56	6
19	24	22	21	8	24	22	50	6
18	25	22	29	7	25	22	44	7
18	26	22	36	7	26	22	37	7
18	27	22	43	6	27	22	30	8
17	28	22	49	6	28	22	22	8
17	29	22	55	6	29	22	14	8
17	30	23	1	5	30	22	6	9
17					31	21	57	

For the Year 1883.

No.	Date	Days of the Month												Total
		1	2	3	4	5	6	7	8	9	10	11	12	
1	Jan	1	2	3	4	5	6	7	8	9	10	11	12	13
2	Feb	1	2	3	4	5	6	7	8	9	10	11	12	13
3	Mar	1	2	3	4	5	6	7	8	9	10	11	12	13
4	Apr	1	2	3	4	5	6	7	8	9	10	11	12	13
5	May	1	2	3	4	5	6	7	8	9	10	11	12	13
6	Jun	1	2	3	4	5	6	7	8	9	10	11	12	13
7	Jul	1	2	3	4	5	6	7	8	9	10	11	12	13
8	Aug	1	2	3	4	5	6	7	8	9	10	11	12	13
9	Sep	1	2	3	4	5	6	7	8	9	10	11	12	13
10	Oct	1	2	3	4	5	6	7	8	9	10	11	12	13
11	Nov	1	2	3	4	5	6	7	8	9	10	11	12	13
12	Dec	1	2	3	4	5	6	7	8	9	10	11	12	13

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TABLE

Of the Suns Declination

For the Year 1684.

Being

Bissextile or Leap-Year.

For the Year 1684.

Days	January		Diff. M.	Days	February		Diff. M.
	D.	M. South			D.	M. South	
1	21	48	10	1	13	57	20
2	21	38	10	2	13	37	20
3	21	28	11	4	13	17	20
4	21	17	11	4	12	57	21
5	21	6	11	5	12	36	21
6	20	55	12	6	12	15	21
7	20	43	12	7	11	54	21
8	20	31	13	8	11	33	21
9	20	18	13	9	11	12	21
10	20	5	13	10	10	51	22
11	19	52	13	11	10	29	22
12	19	39	14	12	10	7	22
13	19	25	15	13	9	45	22
14	19	10	15	14	9	23	22
15	18	55	15	15	9	1	23
16	18	40	15	16	8	38	22
17	18	25	16	17	8	16	23
18	18	9	16	18	7	53	23
19	17	53	16	19	7	30	23
20	17	37	17	20	7	7	23
21	17	20	17	21	6	44	23
22	17	3	17	22	6	21	23
23	16	46	18	23	5	58	23
24	16	28	18	24	5	35	23
25	16	10	18	25	5	12	23
26	15	52	19	26	4	49	24
27	15	33	19	27	4	25	23
28	15	14	19	28	4	2	24
29	14	55	19		3	38	23
30	14	36	19				
31	14	17	19				

Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
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For the Year 1684.

Days	March		Diff. M.	Days	April		Diff. M.
	D.	M.			D.	M.	
	So.	No.			North		
1	3	15	24	1	8	44	22
2	2	51	24	2	9	6	22
3	2	27	23	3	9	28	21
4	2	4	24	4	9	49	22
5	1	40	24	5	10	11	21
6	1	16	24	6	10	32	21
7	0	52	23	7	10	53	21
8	0	29	24	8	11	14	20
9	0	5	24	9	11	34	21
10	0 ^{No.}	19	23	10	11	55	20
11	0	42	24	11	12	15	20
12	1	6	23	12	12	35	20
13	1	29	24	13	12	55	20
14	1	53	24	14	13	15	19
15	2	17	23	15	13	34	19
16	2	40	24	16	13	53	19
17	3	4	23	17	14	12	19
18	3	27	23	18	14	31	18
19	3	50	23	19	14	49	18
20	4	13	24	20	15	7	18
21	4	37	23	21	15	25	18
22	5	0	23	22	15	43	18
23	5	23	23	23	16	1	17
24	5	45	22	24	16	18	17
25	6	8	23	25	16	35	17
26	6	31	22	26	16	52	17
27	6	53	23	27	17	8	16
28	7	16	22	28	17	24	16
29	7	38	22	29	17	40	15
30	8	0	22	30	17	55	15
31	8	22					

For the Year 1684.

Days	May		Diff. M.
	D.	M. North.	
1	18	10	15
2	18	25	15
3	18	40	14
4	18	54	14
5	19	8	14
6	19	22	13
7	19	35	13
8	19	48	13
9	20	1	12
10	20	13	12
11	20	25	12
12	20	37	11
13	20	48	11
14	20	59	11
15	21	10	10
16	21	20	10
17	21	30	10
18	21	40	9
19	21	49	9
20	21	58	8
21	22	6	8
22	22	14	7
23	22	21	7
24	22	28	7
25	22	35	7
26	22	42	6
27	22	48	5
28	22	53	5
29	22	58	5
30	23	3	5
31	23	8	4

Days	June		Diff. M.
	D.	M. North	
1	23	12	4
2	23	16	3
3	23	19	3
4	23	21	3
5	23	24	2
6	23	26	1
7	23	27	1
8	23	28	1
9	23	29	0
10	23	29	0
11	23	29	0
12	23	29	1
13	23	28	1
14	23	27	2
15	23	25	2
16	23	23	3
17	23	20	3
18	23	17	3
19	23	14	4
20	23	10	4
21	23	6	5
22	23	1	5
23	22	56	5
24	22	51	6
25	22	45	6
26	22	39	7
27	22	32	7
28	22	25	7
29	22	18	8
30	22	10	8

For the Year 1684.

Diff. M. 4 3 3 3 2 1 1 1 0 0 0 1 1 2 2 3 3 3 3 4 4 5 5 5 6 6 7 7 7 7 8 8

Days	July		Diff. M.
	D. M.	North.	
1	22	2	0
2	21	53	0
3	21	44	0
4	21	35	0
5	21	26	10
6	21	16	11
7	21	5	11
8	20	54	11
9	20	43	11
10	20	32	12
11	20	20	12
12	20	8	12
13	19	56	13
14	19	43	13
15	19	30	14
16	19	16	14
17	19	2	14
18	18	48	14
19	18	34	15
20	18	19	15
21	18	4	16
22	17	49	16
23	17	33	16
24	17	17	17
25	17	1	17
26	16	45	17
27	16	28	17
28	16	11	17
29	15	54	17
30	15	37	18
31	15	19	

Days	August		Diff. M.
	D. M.	North.	
1	15	1	18
2	14	43	18
3	14	25	19
4	14	6	19
5	13	47	19
6	13	28	19
7	13	9	20
8	12	49	20
9	12	29	20
10	12	9	20
11	11	49	20
12	11	29	20
13	11	8	21
14	10	48	20
15	10	27	21
16	10	6	21
17	9	45	21
18	9	23	22
19	9	2	21
20	8	40	22
21	8	18	22
22	7	56	22
23	7	34	22
24	7	12	22
25	6	50	23
26	6	27	23
27	6	5	22
28	5	42	23
29	5	19	23
30	4	57	23
31	4	34	

For the Year 1684.

Days	September		Diff. M.	Days	October		Diff. M.
	D.	M.			D.	M.	
	No.	So.			Source		
1	4	11	23	1	7	28	22
2	4	48	23	2	7	50	23
3	3	25	24	3	8	19	22
4	3	1	23	4	8	35	22
5	2	38	23	5	8	57	22
6	2	15	23	6	9	19	22
7	1	52	24	7	9	41	22
8	1	28	23	8	10	5	22
9	1	5	24	9	10	25	21
10	0	41	23	10	10	46	22
11	0	18	24	11	11	8	21
12	0	6	23	12	11	29	21
13	0	29	24	13	11	50	21
14	0	53	23	14	12	11	21
15	1	16	24	15	12	32	20
16	1	40	23	16	12	52	20
17	2	3	24	17	13	12	20
18	2	27	23	18	13	32	20
19	2	50	24	19	13	52	20
20	3	14	23	20	14	12	19
21	3	37	23	21	14	31	19
22	4	0	24	22	14	50	19
23	4	24	23	23	15	9	19
24	4	47	23	24	15	28	18
25	5	10	23	25	15	46	18
26	5	33	23	26	16	4	18
27	5	56	23	27	16	22	18
28	6	19	23	28	16	40	17
29	6	42	23	29	16	57	17
30	7	5	23	30	17	14	17
				31	17	31	

For the Year 1684.

Days	November		Diff. M.		Days	December		Diff. M.
	D.	M.				M.	D.	
	South					South		
1	17	47	16		1	23	9	4
2	18	3	16		2	23	13	4
3	18	19	16		3	23	17	3
4	18	35	15		4	23	20	3
5	18	50	15		5	23	23	2
6	19	5	14		6	23	25	2
7	19	19	14		7	23	27	1
8	19	33	14		8	23	28	1
9	19	47	14		9	23	29	0
10	20	1	13		10	23	29	0
11	20	14	12		11	23	29	0
12	20	26	13		12	23	29	1
13	20	39	12		13	23	28	2
14	20	51	11		14	23	26	2
15	21	2	11		15	23	24	2
16	21	13	11		16	23	22	3
17	21	24	10		17	23	19	3
18	21	34	10		18	23	16	4
19	21	44	10		19	23	12	4
20	21	54	9		20	23	8	5
21	22	3	8		21	23	3	5
22	22	11	8		22	22	57	5
23	22	19	8		23	22	52	6
24	22	27	7		24	22	46	7
25	22	34	7		25	22	39	7
26	22	41	7		26	22	32	8
27	22	48	6		27	22	24	8
28	22	54	5		28	22	16	8
29	22	59	5		29	22	8	9
30	23	4	5		30	21	59	9
					31	21	50	

[illegible]

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Height of the Pole or Latitude	Difference of Meridians	Name of Place
0	0	Paris
1	1	Amsterdam
2	2	London
3	3	Brussels
4	4	Antwerp
5	5	Brabant
6	6	Paris
7	7	Amsterdam
8	8	London
9	9	Brussels
10	10	Antwerp
11	11	Brabant
12	12	Paris
13	13	Amsterdam
14	14	London
15	15	Brussels
16	16	Antwerp
17	17	Brabant
18	18	Paris
19	19	Amsterdam
20	20	London
21	21	Brussels
22	22	Antwerp
23	23	Brabant
24	24	Paris
25	25	Amsterdam
26	26	London
27	27	Brussels
28	28	Antwerp
29	29	Brabant
30	30	Paris
31	31	Amsterdam
32	32	London
33	33	Brussels
34	34	Antwerp
35	35	Brabant
36	36	Paris
37	37	Amsterdam
38	38	London
39	39	Brussels
40	40	Antwerp
41	41	Brabant
42	42	Paris
43	43	Amsterdam
44	44	London
45	45	Brussels
46	46	Antwerp
47	47	Brabant
48	48	Paris
49	49	Amsterdam
50	50	London
51	51	Brussels
52	52	Antwerp
53	53	Brabant
54	54	Paris
55	55	Amsterdam
56	56	London
57	57	Brussels
58	58	Antwerp
59	59	Brabant
60	60	Paris

TABLE

Of the Differences of Meridians

Of the most considerable Places

In the whole World,

In respect of *P A R I S*,

With the Height of the Pole or Latitude of the same places.

The Differences of Meridians.

Names of places.	Differences of Meridians		Height of the Place or Latitude	
	H.	M.	D.	M.
Abbeville	0	1 add	50	0
Agra	5	26 subst.	26	0
Aiz Prov.	0	14	43	33
Aleppo	2	50	36	46
Alexandria <i>Æ.</i>	2	13	30	58
Amiens	0	0	49	55
Amsterdam	0	14	52	21
Angers	0	11 add	47	32
Antwerp	0	15 subst.	51	12
Arles	0	11	43	36
Arras	0	2	50	9
Archangelo	2	49	65	30
Athens	1	35	7	40
Avignon	0	11	43	52
Bayonne	1	15 add	43	30
Blois	0	3	47	46
Bordeaux	0	10	44	50
Bourges	0	0	47	20
Boulogne It.	0	40 subst.	44	20
Brest	0	28 add	48	23
Brussels	0	15 subst.	50	48

Of the Differences of Meridians.

Names of Places	Differences of Meridians		Height of the Pole or Latitude	
	H	M.	D.	M.
Cadis	0	37 add	36	16
Caen	0	9	49	11
Callais	0	2	50	57
Camboge	6	53 subst.	11	20
Cambray	0	3	50	5
Cap. of b. Es.	1	11	34	32 South
Cap-Verd	1	34 add	14	27 North
Chartres	0	4	48	30
Constantinop.	2	0 subst.	42	56
Copenhagen	0	42	55	43
Cracovia	1	14	50	10
Dantzick	1	12 subst.	54	22
Dieppe	0	4 add	49	56
Dijon	0	12 subst.	47	28
Dole	0	13	47	20
Doway	0	3	50	15
Dunkerke	0	0	51	1 1/2
Ererare	0	41 subst.	44	54
Isle de Fer.	1	38 add	28	10
Fez	0	31	33	10
La Fleche	0	10	47	42

The Differences of Meridians. 30

Names of Places.	Differences of Meridians		Height of the Pole or Latitude	
	H.	M.	D.	M.
Florence	0	40 ¹ subst.	43	41
Francfort M.	0	27 ⁵ 7 ⁸	50	4
Fribourg	0	25 9	48	16
Gaunt	0	16 ¹ 27	51	2
Geneva	0	17 8	46	20
Goa	0	58 11	115	30
Grenoble	0	35 ⁵ 4 ⁸	45	11
Hambourgh	0	11 ⁴ 0	53	43
Havre de Gr.	0	8 add	49	7
Hierusalem	2	36 subst.	32	0
Hispam	4	16	36	14
Rechelle	0	13 ⁵ add	46	11
Leiden	0	11 ¹ subst.	52	12
Lima Per.	5	31 add	12	20 South
Lisbon	0	56 8	38	40 North
London	0	8 0	51	32
Loudun	0	8	48	0
Lovain	0	13 ¹ subst.	50	50
Luques	0	37 ¹ 8 ⁸	43	40
Luxembourg	0	20 1 ⁸	49	38
Lyons	0	11 0	45	46

The Differences of Meridians.

Names of Places.	Differences of Meridians		Height of the Pole or Latitude	
	D.	M.	D.	M.
Macon	0	11 subft.	46	20
Madrid	0	25 add	40	10
Malta	0	53 subft.	35	40
Malaca	6	45	2	20
S. Malo	0	18 add	48	39
Mantove	0	37 subft.	45	1
Marseilles	0	14	43	15
Martinique	4	15 add	14	14
Meaux	0	2 subft.	48	56
Messina	0	58	38	21
Metz	0	19	49	10
Messique	7	8 add	20	30
Millan	0	31 subft.	45	15
Modene	0	38	44	39
Monaco	0	23	43	39
Montpellier	0	6	43	37
Munick	0	50	48	58
Namur	0	13 subft.	50	26
Nancy	0	19	48	39
Nantes	0	16	46	13
Naples	0	56	41	5
Narbonne	0	3	43	6
Nevers	0	2	47	19
				N

The Differences of Meridians.

Names of Places.	Differences of Meridians.		Height of the Pole or Latitude	
	H.	M.	D.	M.
Ostend	0	3 subst.	51	16
Orleans	0	2 add.	47	54
Ormuz	4	0 subst.	27	35
Paris	0	0	48	51
Parma	0	36 subst.	44	45
Pavia	0	31	44	58
Padoua	0	42	45	31
Pexing Chi.	7	45	40	0
Perigeux	0	9 add.	45	34
Perpignan	0	5 subst.	45	52
Pernanibouc.	2	57 add.	7	40 South
Perouge	0	44 subst.	42	56 North
Pisa	0	38	43	9
Plaisance	0	33	44	53
Poictiers	0	6 add.	47	7
Porto-belo.	5	41	9	55
Prague	0	58 1/2 subst.	50	40 1/2
Quebec	4	36 add.	47	0
S. Quentin.	0	6 subst.	49	46
Rennes	0	16 add.	47	58
Rhemes	0	9 subst.	49	12
Riga	1	31	56	52

The Differences of Meridians.

Names of Places.	Differences of Meridians		Height of the Pole or Latitude	
	H.	M.	D.	M.
Rome	0	47 subst.	41	54
Rouen	0	4 add	49	27 ¹ / ₂
Saintes	0	11 ¹ / ₂ add	45	38
Savone	0	28 subst.	44	18
Sienné	0	41	43	11
Siras	3	42	34	14
Smirna	2	56	38	22
Strasbourg	0	24	48	31
Stettin	0	54	53	34
Stockholm	1	7	59	30
Tangier	0	56 add	35	25
Tholouse	0	2	43	29
Toleda	0	26	39	52
Tours	0	6	47	35
Tournay	0	5 subst.	50	32
Tutin	0	25	44	9
Toulon	0	17	43	
Valencienne	0	4 subst.	50	20
Verdun	0	38	45	33
Vienna	1	2	48	22

The History of the

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ADVERTISEMENT.

I Have begun to travel for the Impression of an intire Work of the *Conique Sections*, where you shall find not only all the most excellent things has been discovered in this part of the *Mathematicks*, but also a great number of new Properties which I have discovered, and whereof I have published every one upon different occasions.

The Method of the Principal Demonstration is particular to my self, and I have used it to a-bridge very much this whole Work; I have given an Essay of this New Method, which I caused to be Printed, concerning the *Conique Sections* in the Year 1675.

See here in few words the order of these Books and what they contain.

The First Book contains the *Lemmas* that are necessary for this Method.

The Second contains the original of the *Three Conique Sections*, with the Properties of their Diameters, and all that which depends on a Line cut harmonically or into two equal parts, the original of the *Asymptotes*, and in the end divers Problems depending on these Principles.

The

The Third explains the relation of the *Ordinate Lines* to their *Diameters*, with the *Rect-angles* of the parts of the *Diameters*, *Parameters*, and that which concerns the *Tangents*.

The Fourth makes that appear which is most considerable touching the *Asymptotes*.

The Fifth Book is fill'd with near 40 most curious Propositions upon the *Conique Sections*.

In the Sixth Book is treated of equal and like *Sections*.

The Seventh is continued all along on that which is called the least and greatest.

The Eighth treats of the *Foci* of the *Three Sections*.

In the Ninth Book is taught certain, most plain, and most useful Methods for the Description of these *Sections*.

Then I shew you in an Appendix after what manner we ought to resolve the *Conique Sections* that have for their *Bases*, *Paraboles*, *Hyperboles* and *Elipses*, and also all *Cylinders* of the same *Species*.

Afterwards I demonstrate that by the same Method, and without difficulty, we may demonstrate all the Properties of the *Conique Sections* of whatsoever compounded kind they are, I draw them from their *Cones* or *Pyramids*, which I distinguished by *Cones* of the first kind, of the second kind, of the third kind, &c. These sorts of *Cones* have for their *Bases* *Circles* of all these kinds, and then having

ving explained after what manner we may render
all the *Lemma's* of the first Book universal, I ex-
plain in the same manner that which is the Se-
cond Book, and in one part of the following the
Sections of all *Cones* of any kind.

The Demonstrations are the same as for the Se-
ctions of the first *Cone*, which is that which has the
Circle of the first kind for its base : After that I
see not any thing that we can desire more univer-
sal, nor more easie on this same *Atiere*.

The

The Approbation.

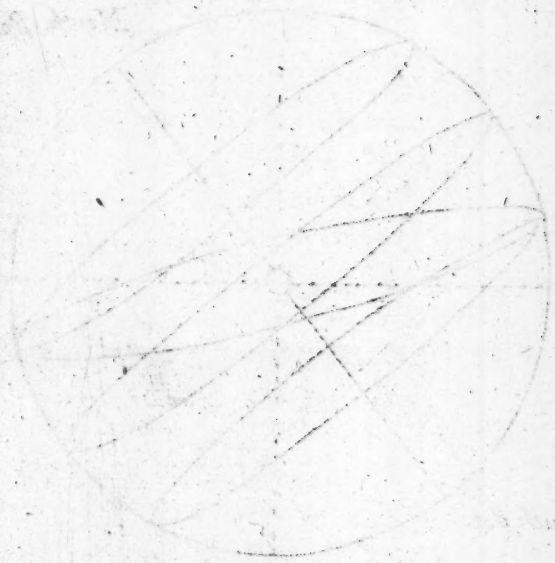
THis Treatise of the Gnomiques,
or the Art of Drawing of Sun
Dials, composed by Mr. De la Hire,
 hath been read to the Assembly of the
Academy Royal of Sciences, made
the 9th of May 1682.

J. B. Du Hamel Secretary of the
Academy of Sciences.

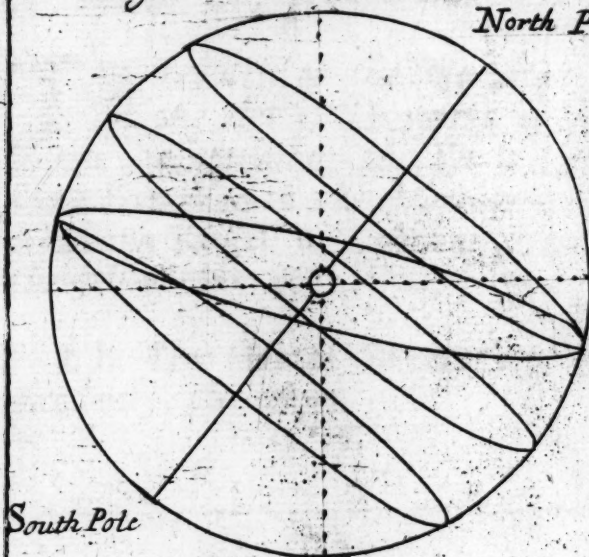
THe Approbation seen, per-
mitted to be Imprinted,
made the 23th of May 1682.

DE LA REYNIE.

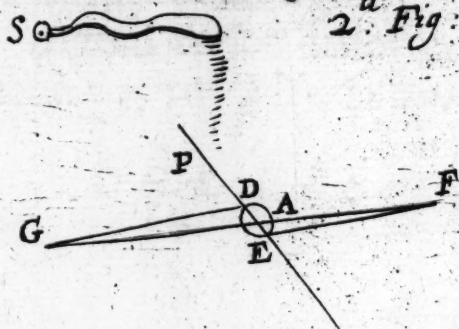
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1st Figure Zenith North Pole



2^d Fig.



3^d Fig.



Pole



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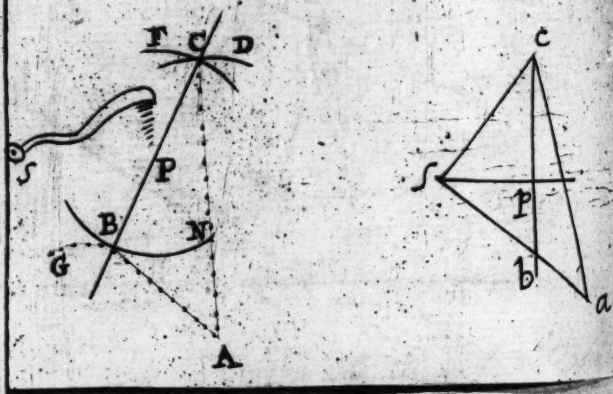
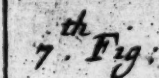
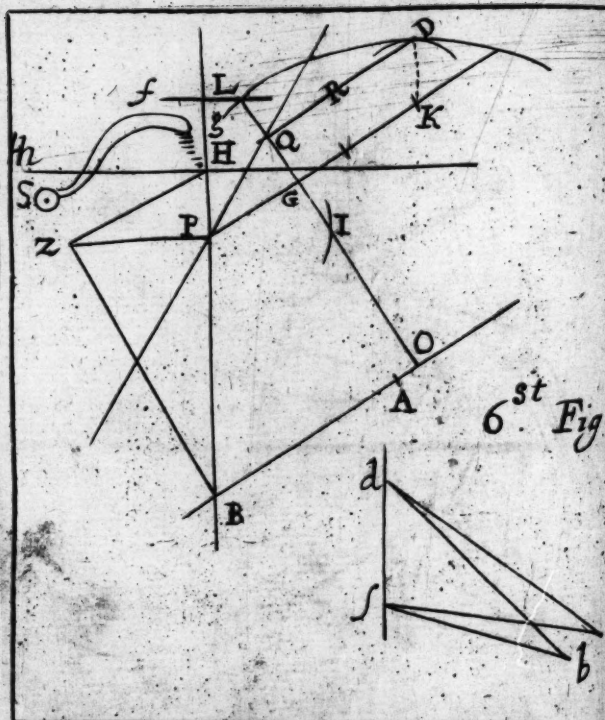
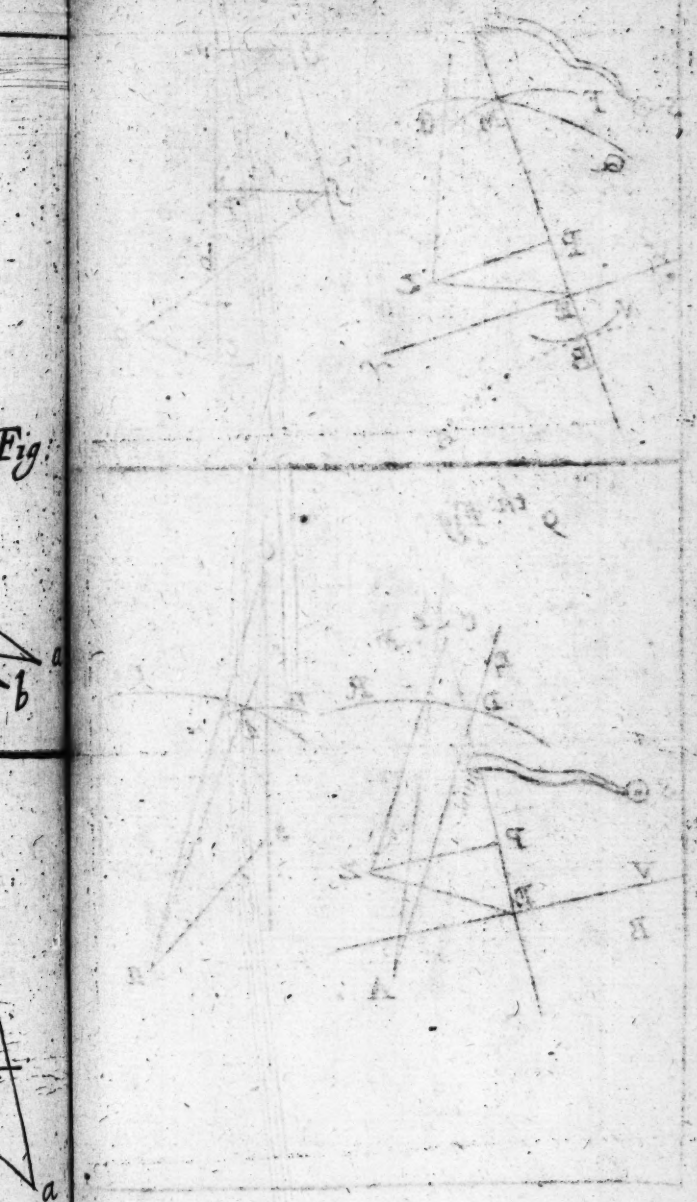


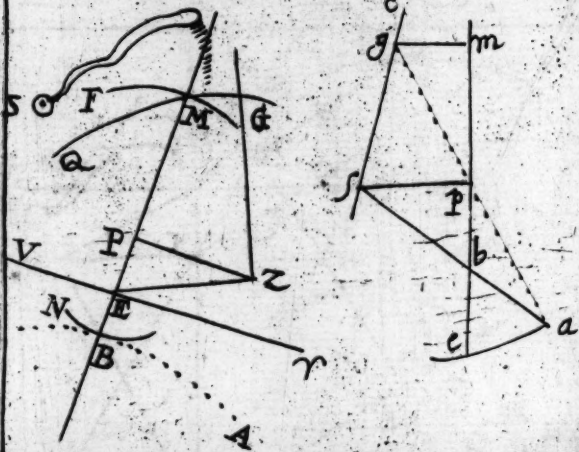
Fig.

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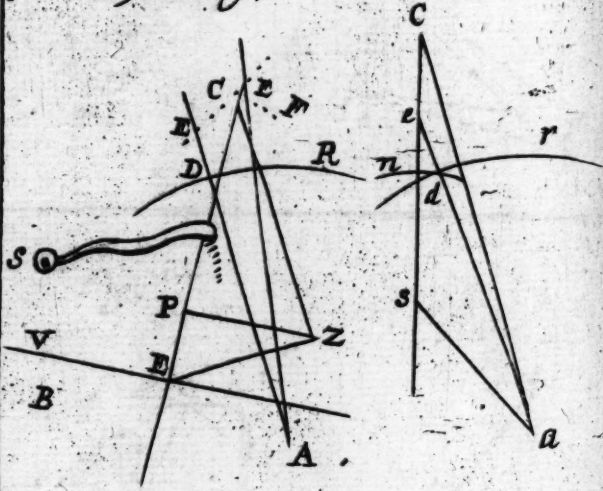
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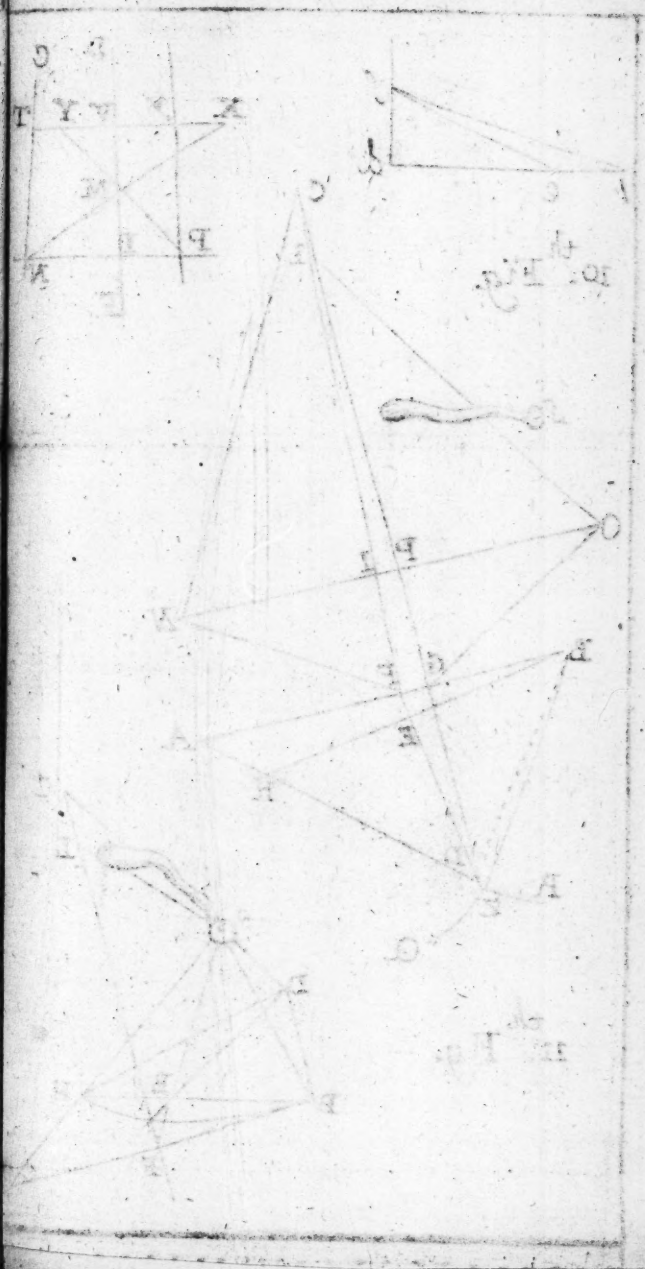


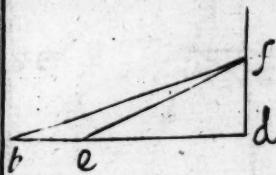
8th Fig:



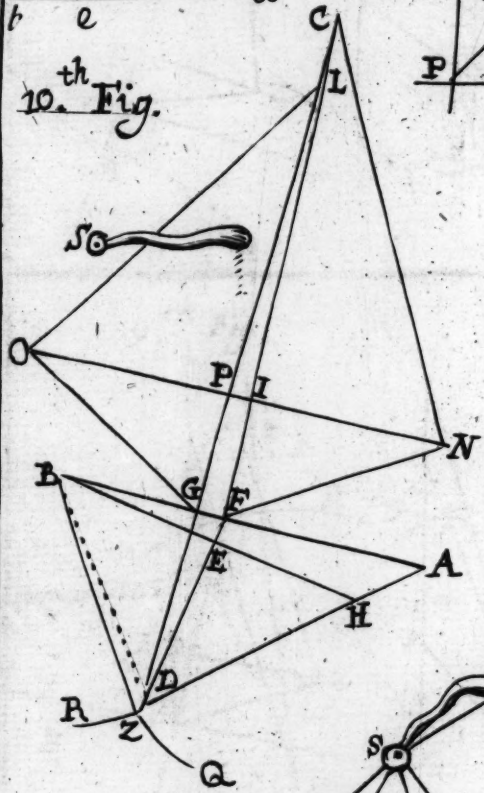
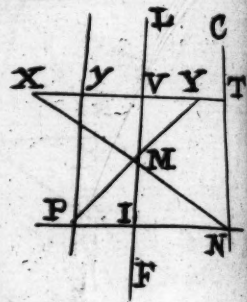
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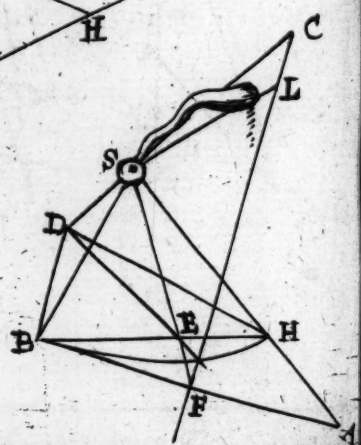




10.th Fig.



11.th Fig.

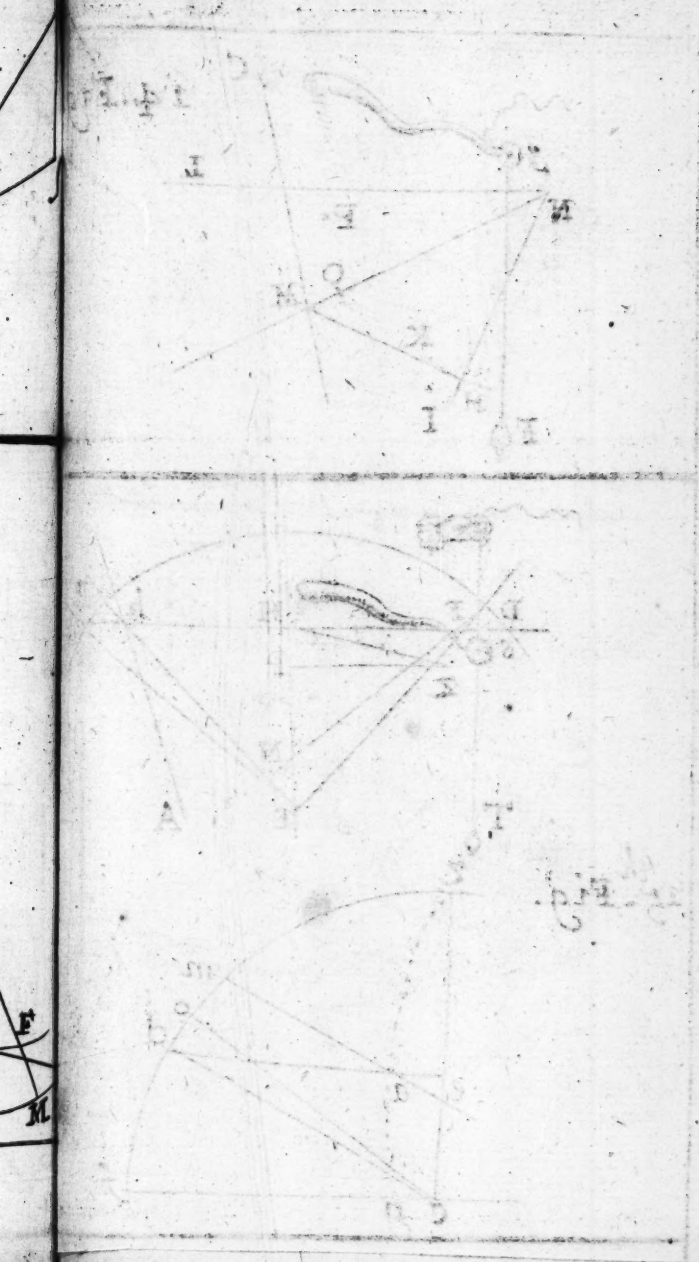


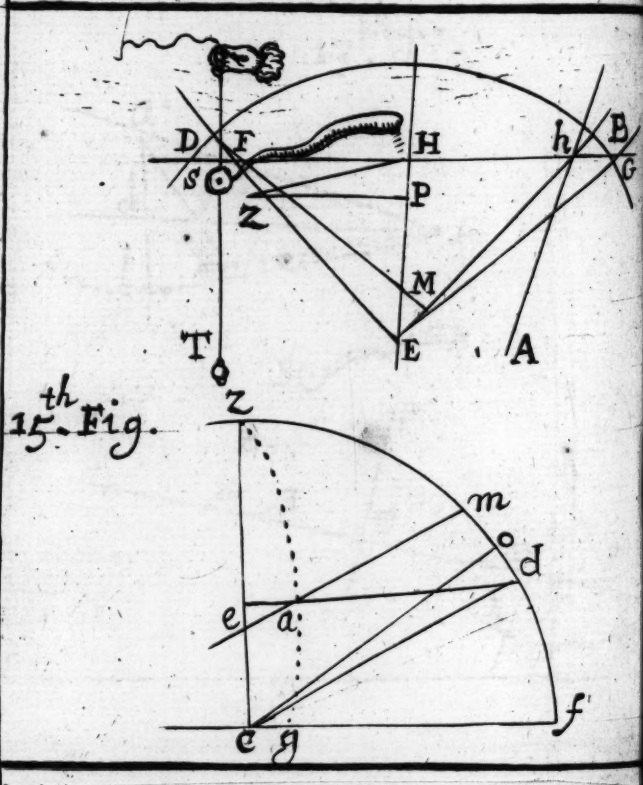
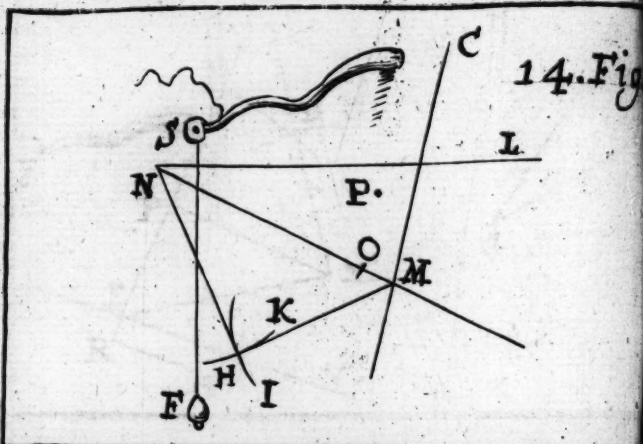
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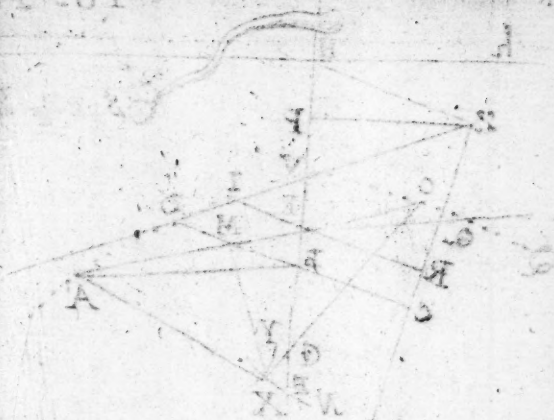
A







Fig



B
G

Fig 2

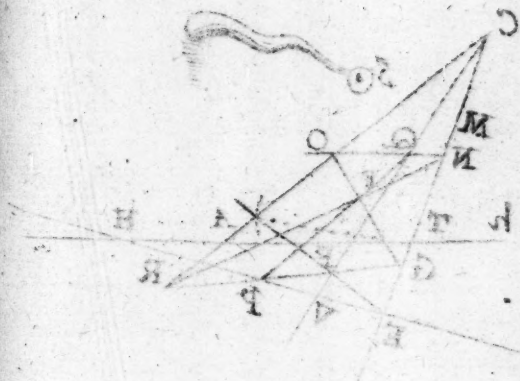
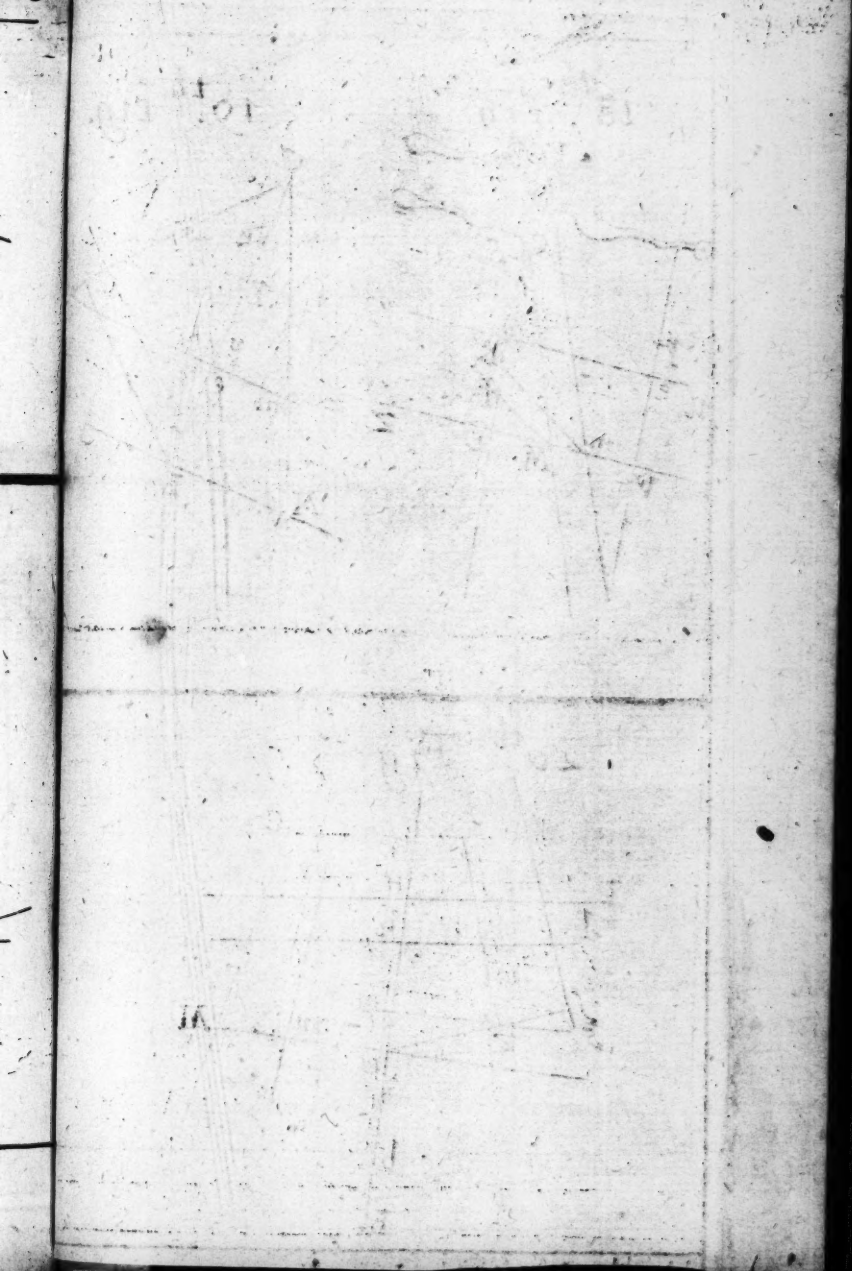
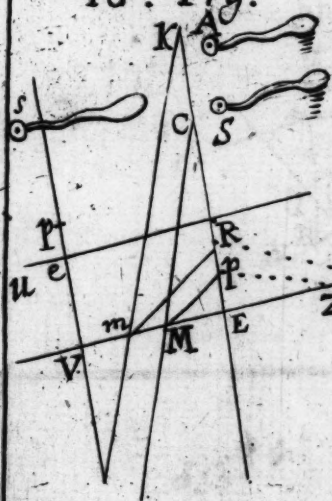


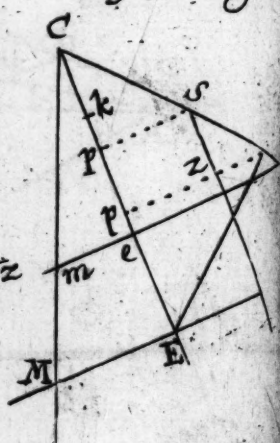
Fig.



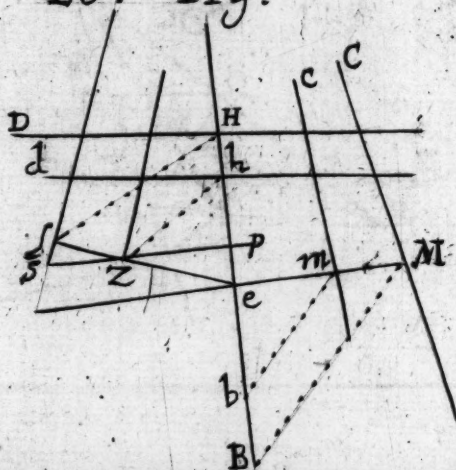
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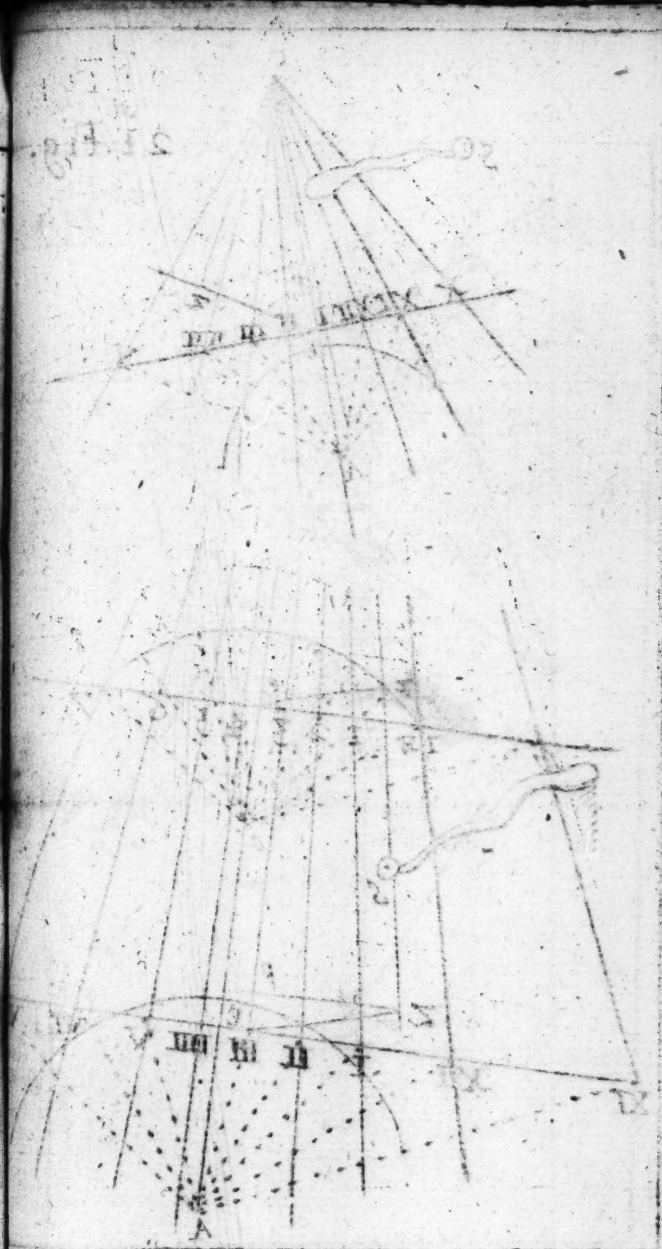
19th Fig.

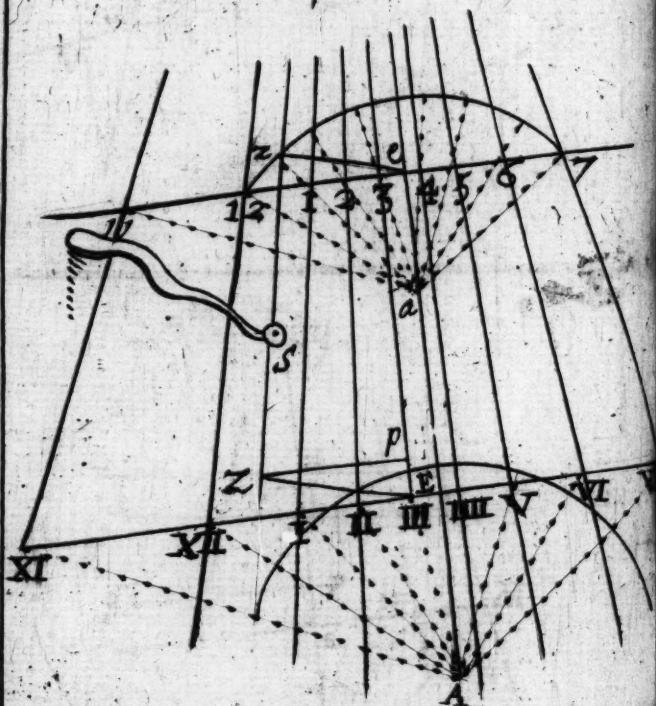
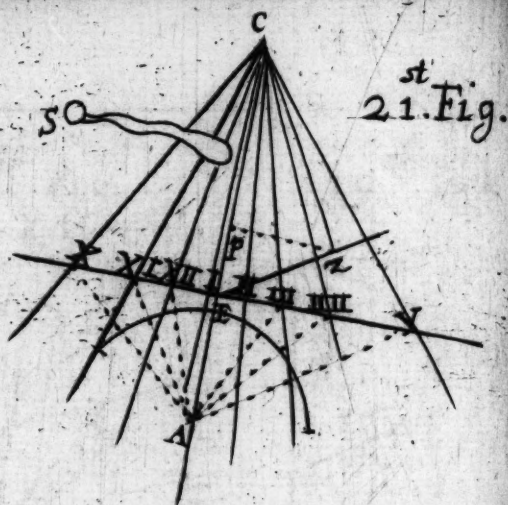


20th Fig.

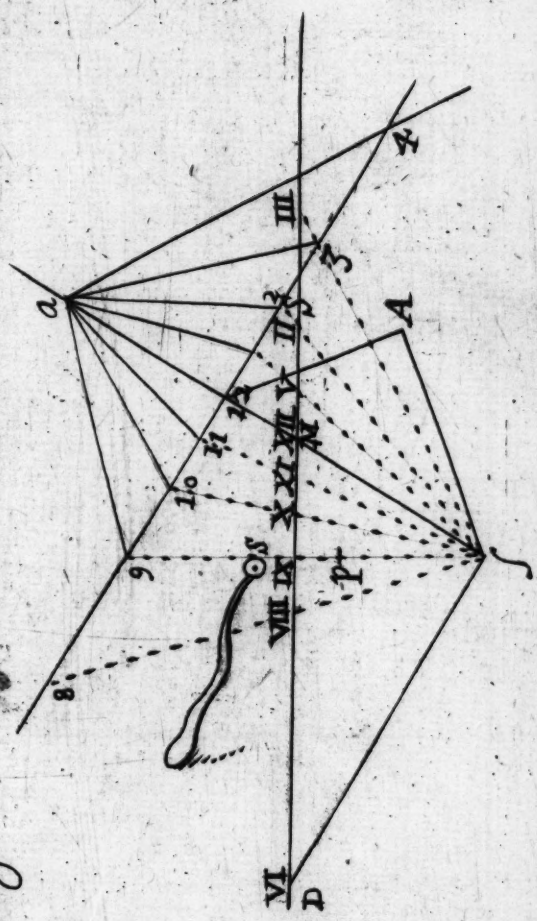


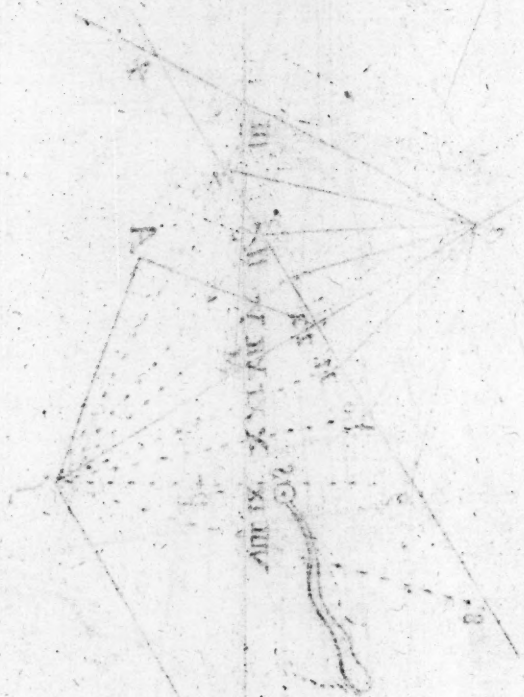
9.



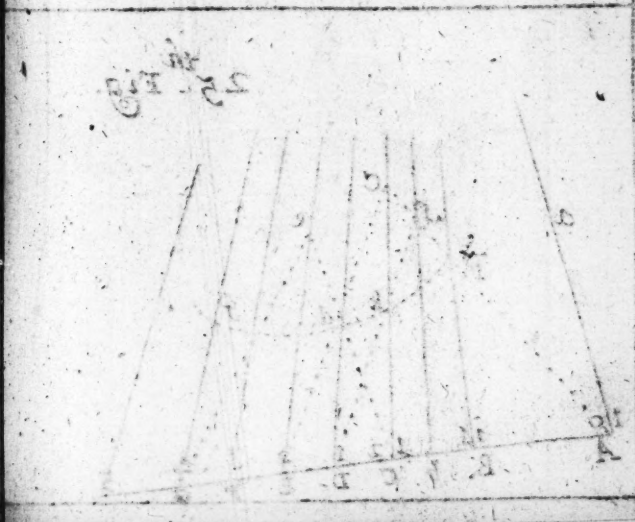
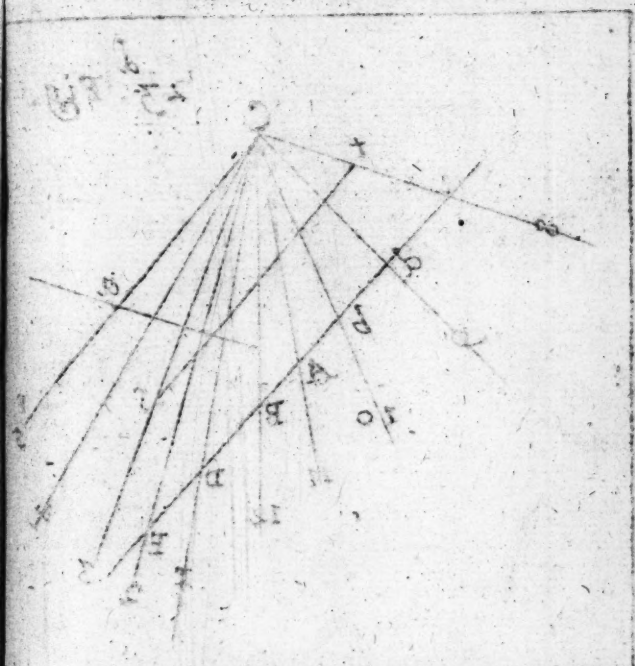


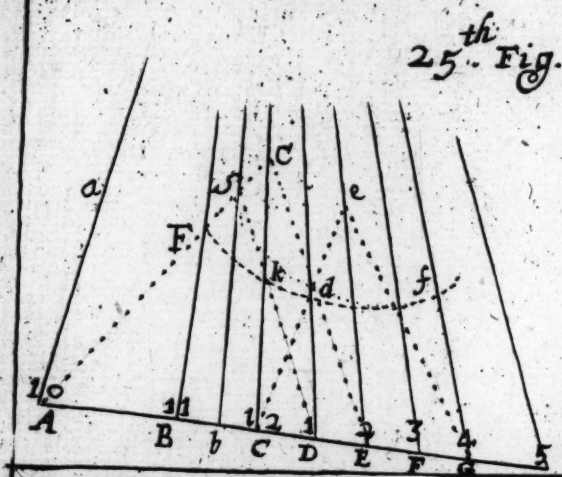
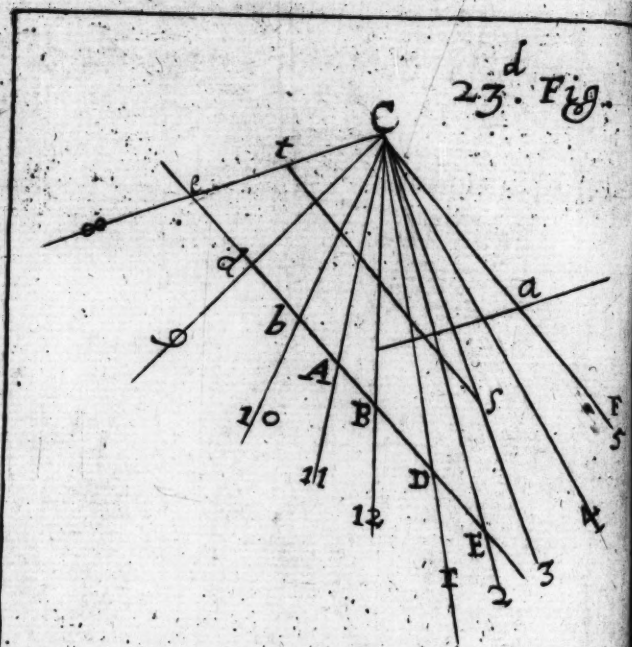
22^d. Fig.





ss. l. d.





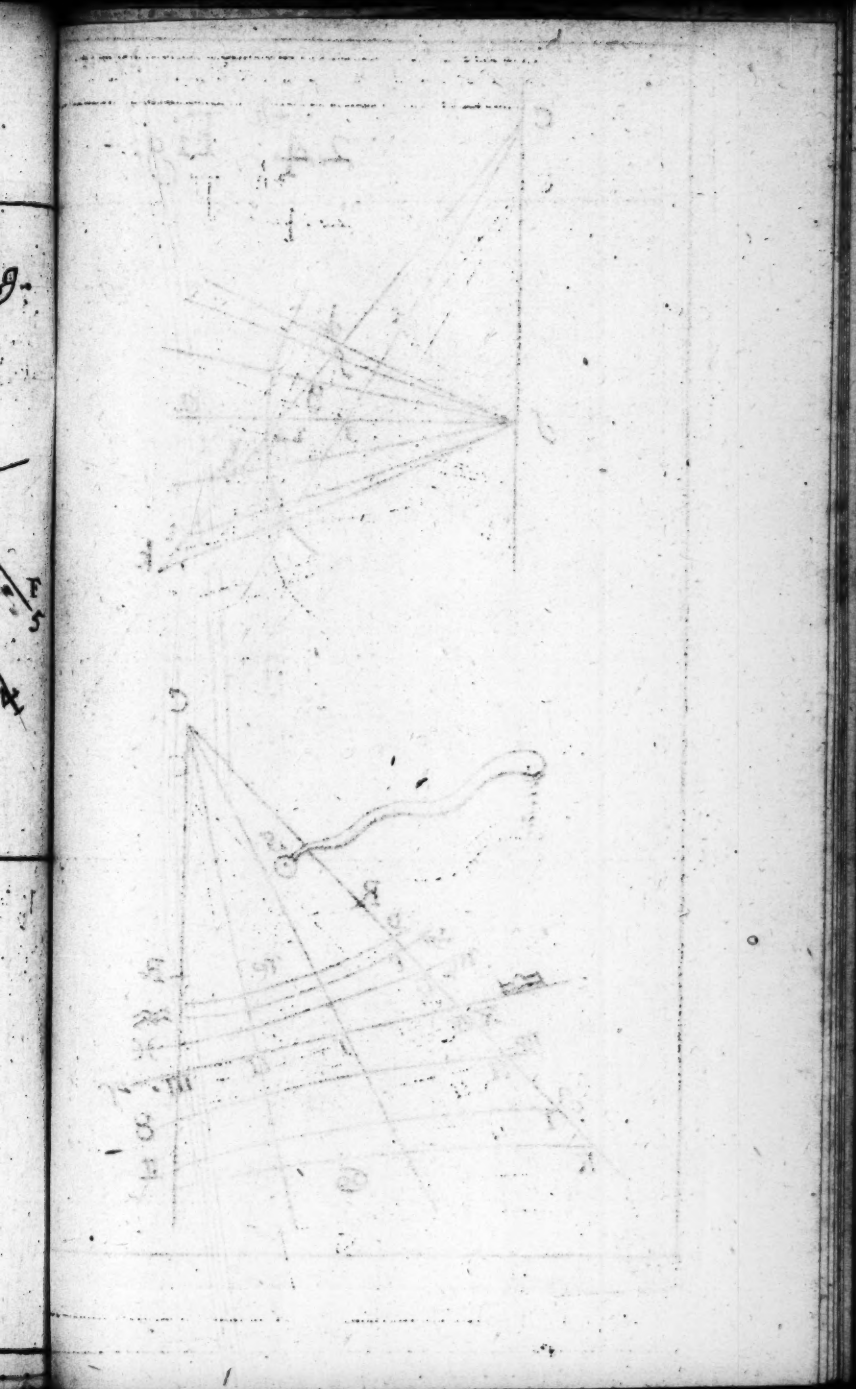


Fig. 26.

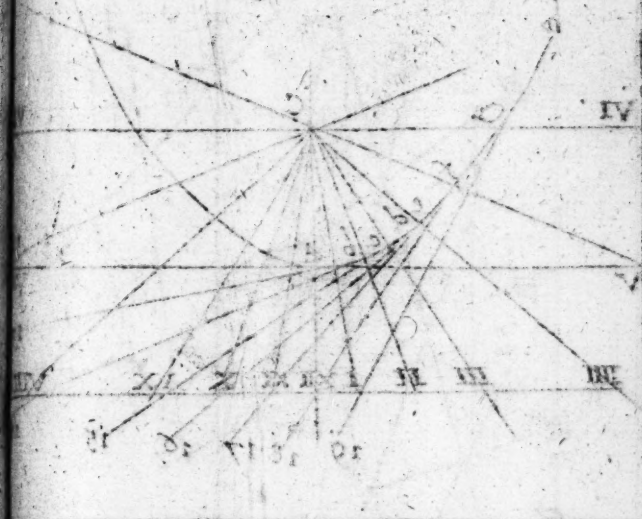
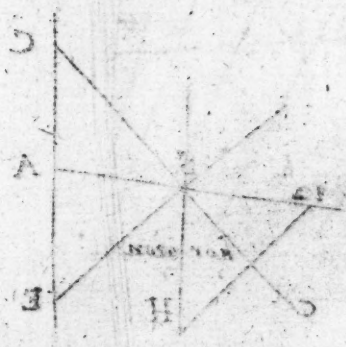
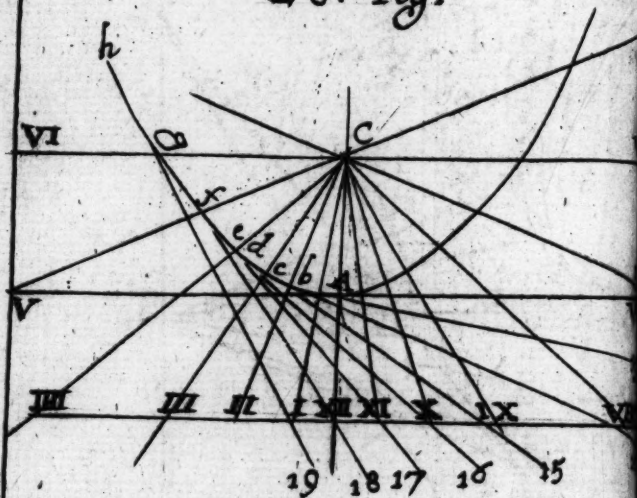


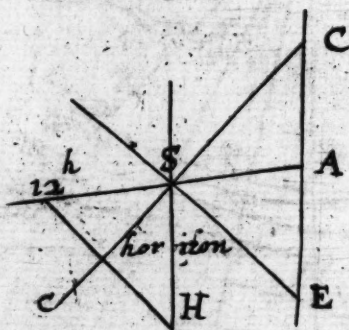
Fig. 27.



26th Fig.

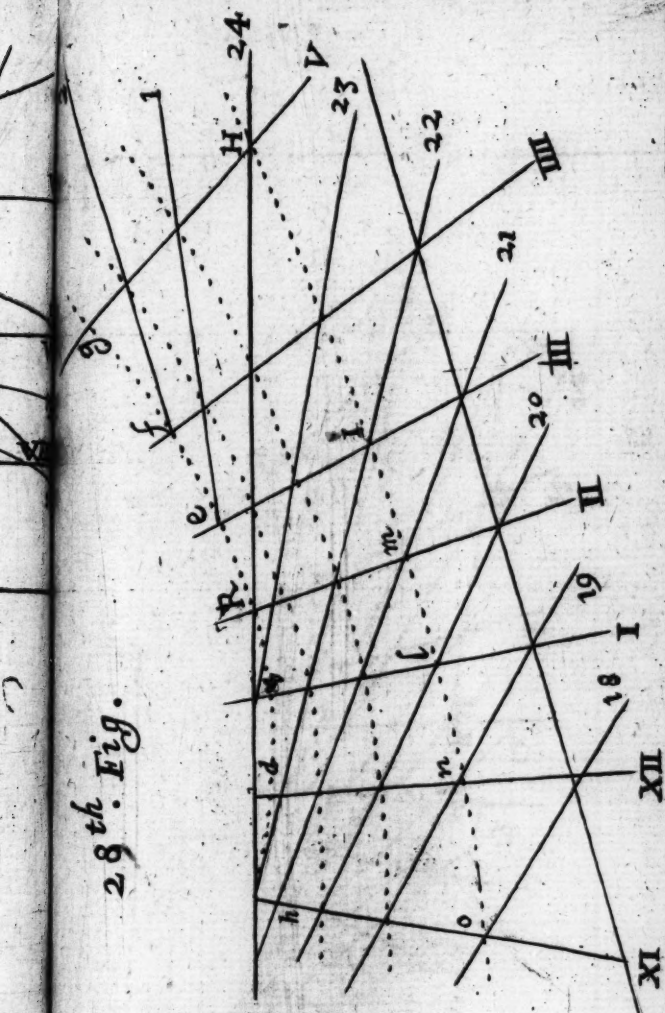


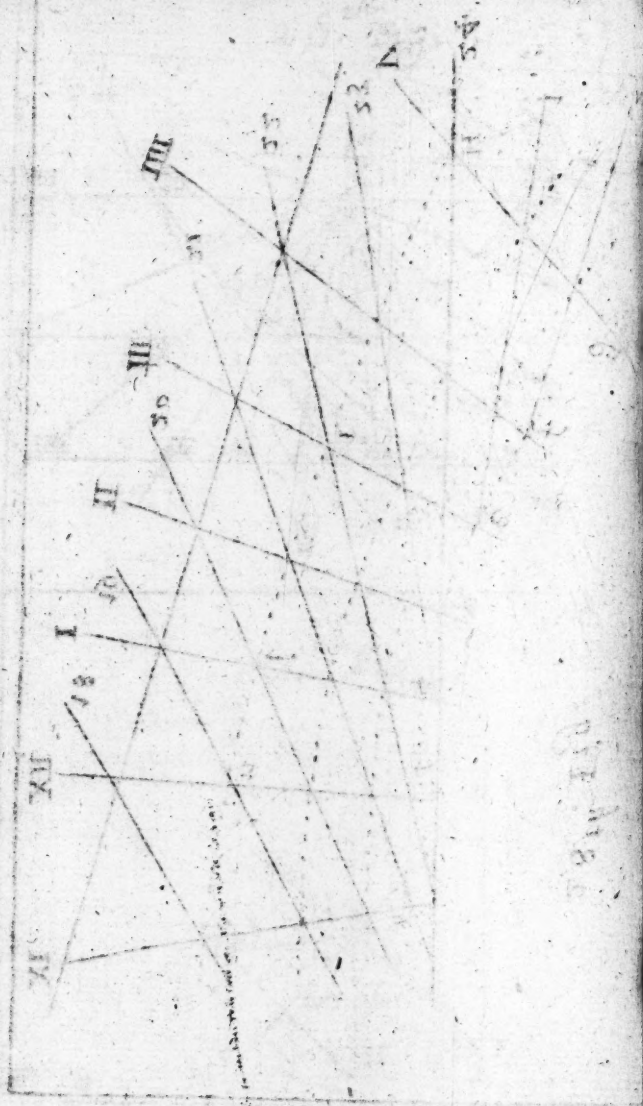
27th Fig.



28th Fig.

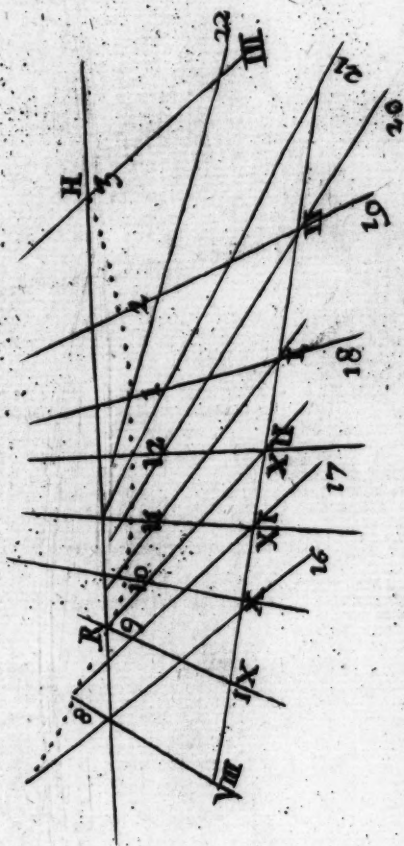
28th Fig.

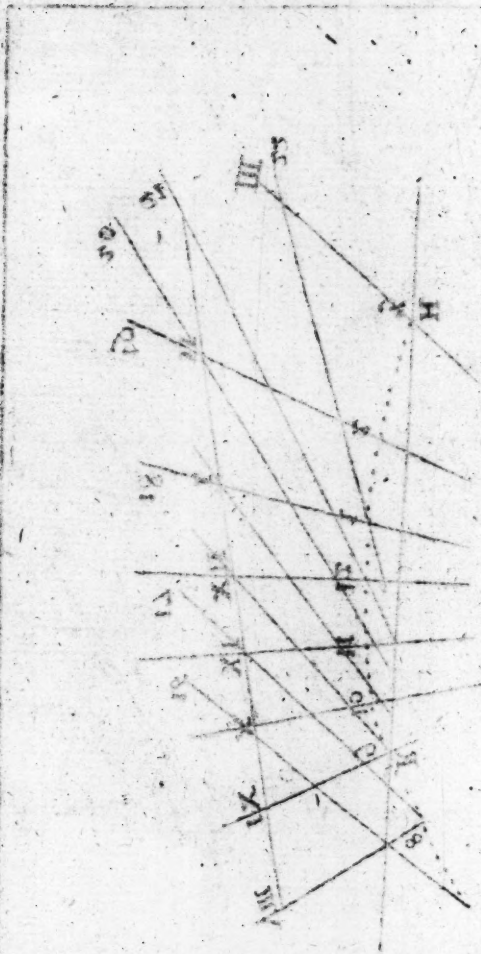




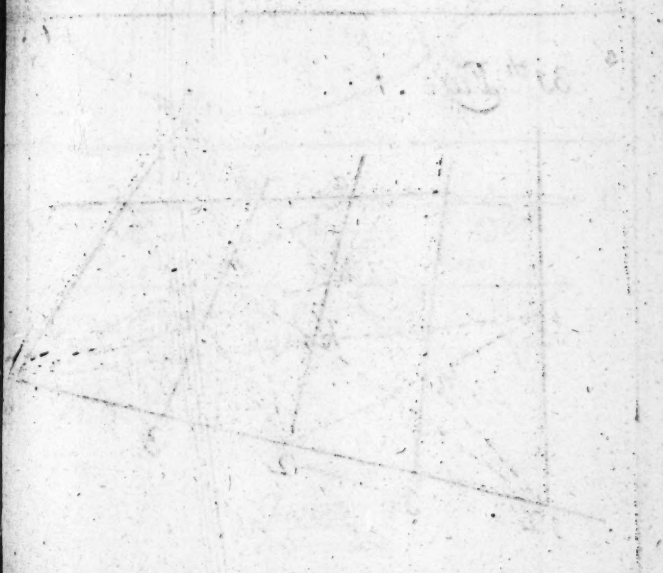
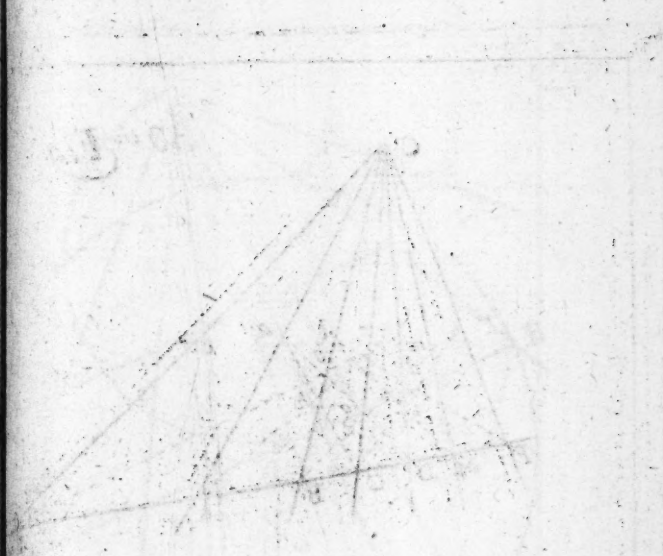
29th Fig.

39th Fig.

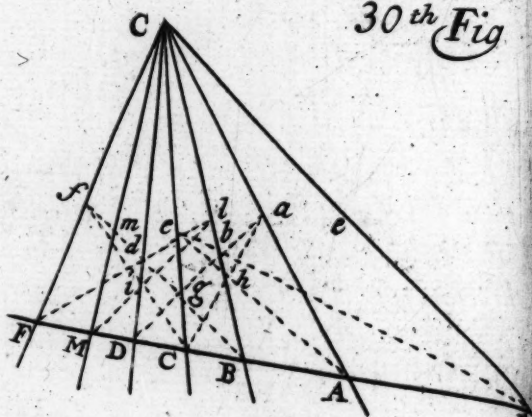




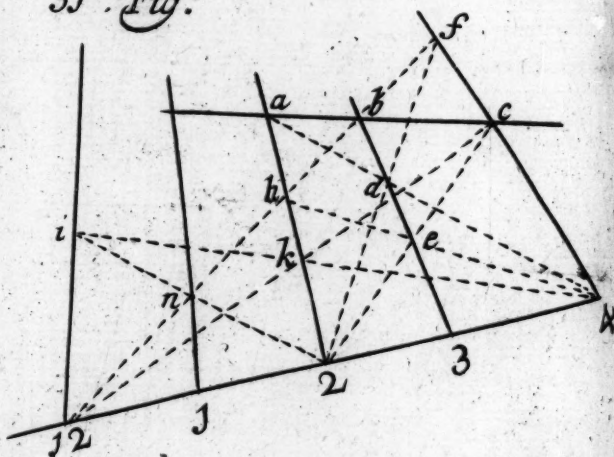
30. 11. 11.



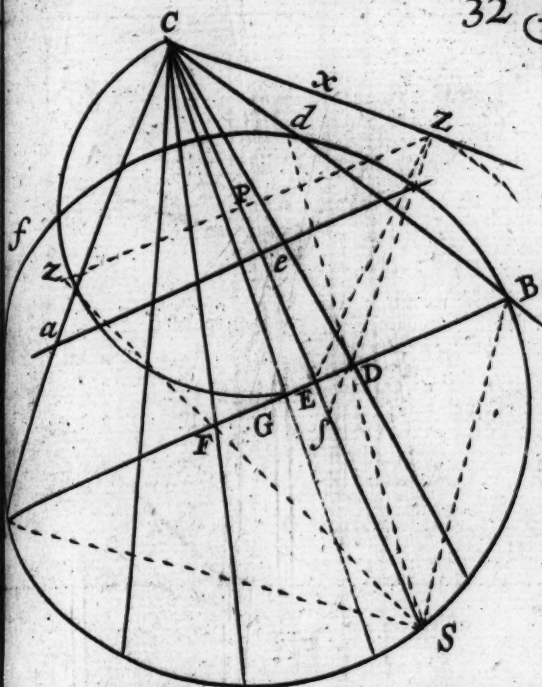
30th Fig



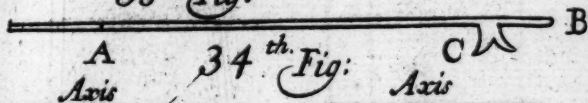
31th Fig:



32th Fig:



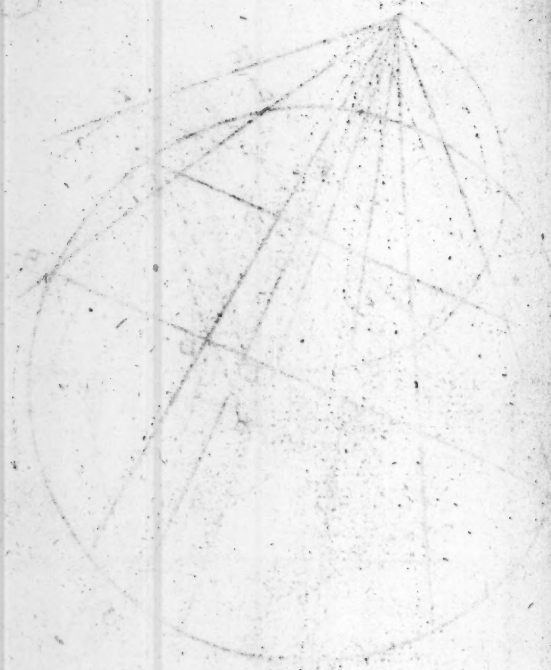
33th Fig:



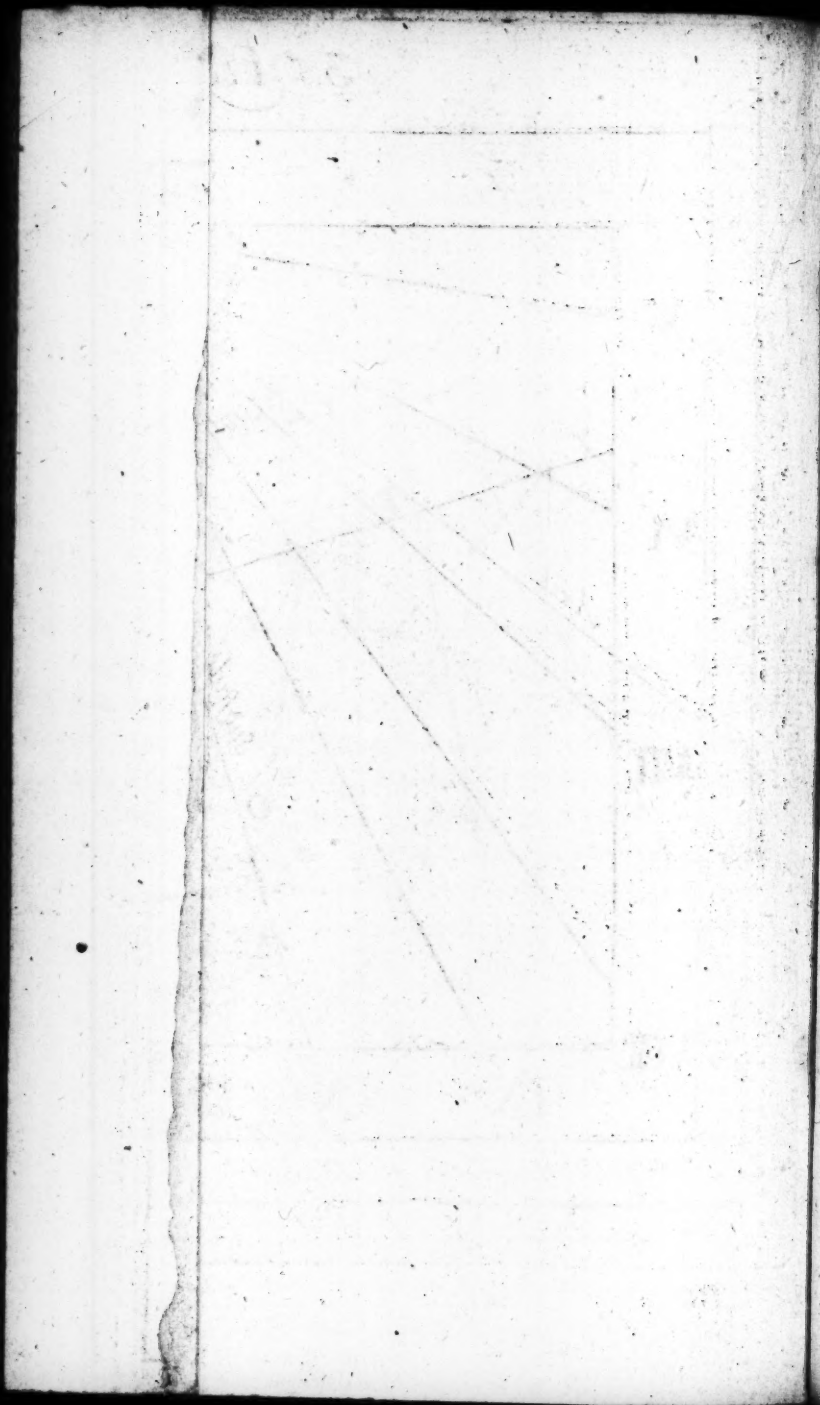
34th Fig:



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THis Geometrical way of Mr. De la Hire for Drawing Sun-Dials on Fixed Planes is universal. The Instruments that are used to perform the Practice thereof are only a Plain Scale and a Pair of Compasses for finding the Substyle and Equinoctial Lines on the given Plane, with the situation of the Axis of the Style in respect of the said Plane, the Observations being made by the Sun from several Shadows of the Point of a Fixed Style set upon the said Plane. And by a Level and Plumb Line for finding the situation of the Meridian and Horizontal Lines of the Place on the Plane of the Dial, which Practice of Dialling he chiefly demonstrates from the Intersections of Planes, the Doctrine whereof is contained in the 11th Book of Euclid's Elements. And because he has given no particular Example of this general way of Dialling in this Treatise, I have thought good to add this Example following:

A particular Example of this General Method of Dialling Illustrated by Numbers.

There is a Style fix'd on a Fixed Plane, whose perpendicular height is SP 1 Inch 64 parts, or 164 parts. Now, the Sun shining, if there be taken Two Points of Shadow of the top of the Style S on the said Plane, as A and B, on that day when the Sun's Declination is 21 deg. 40 min. South, and that PB the Distance of the Point B,

A

from

from P the foot of the *Style* may be 55 parts, and the Distance of the Point A from P may be P A 366 parts, and that the Distance of the Point A from B may be B A 415 parts.

Now from these things given, it is required to find the situation of the *Substylar* and *Equinoctial Lines*, and to find the *Center* of the *Dial* on the said *Plane*, and to determine the position of the *Axis* of the *Dial* by *Calculation*.

[The Geometrical Operation of this is to be found in the Sixth Chapter, page 16th of this *Treatise*, and in the 5th Figure.]

First in the Right-angled Triangle S P B.

There is given P S 164 parts, the perpendicular height of the *Style*, and P B 55 parts the distance of the Point of Shadow B from P the foot of the *Style*;

To find the Vertical Angle P S B, and the Hypotenuse S B, which is the length of the Ray of the Sun from the top of the *Style* S to the Point of Shadow B,

The Proportion is as S P 164 parts,
Is to P B 55 parts

So is the Radius,

To the Tang. of the Angle P S B 18 d. 32 m.

And

And as the Sine of the Angle PSB 18 d. 32 min.

Is to PB 55 parts,

So is the Sine of 90 deg.

To SB 173 parts.

Secondly, In the Right-angled Triangle S P A,

There is given the perpendicular PS 164 parts

And the Base P A 366 parts,

To find the *Vertical Angle* PSA and the *Hypotenuse* SA.

The Proportion is, As S P 164 parts.

Is to P A 366 parts.

So is the Radius

To the Tangent of the Angle PSA 65 d. 52 m.

And as the Sine of the Angle PSA 65 d. 52 m.

Is to P A 366 parts,

So is the Sine of 90 degrees,

To SA 401 parts.

Thirdly, in the Obliquangular Triangle *fdb.*

There is given the two sides *fb* 173 parts and *fd* 100 parts, taken at pleasure, and the contained Angle *dfb* 68 deg. 20 min. the Complement of the Suns Declination.

A 2

To

To find the other two Angles $\angle dbb$ and $\angle sbd$,
and the third side db .

From --- 180 deg. 00 min.

Take the given Angle $\angle fsb$ 68 20

And there shall remain --- 111 deg. 40 min.

The half whereof is --- 55 deg. 50 min.

Then the greater side is sb 173 parts,
And the less side is sd 100 parts.

Then as the Sum of the sides 273 parts,
Is to the difference of the sides 73 parts:
So is the Tangent of 55 deg. 50 min.

To the Tangent of 21 deg. 30 min.

The Sum is $\angle dbb$ 77 deg. 20 min.

The Difference is $\angle sbd$ 34 deg. 20 min.

Then as the Sine of the Angle $\angle sbd$ 34 d. 20 m.
Is to the Sine of the Angle $\angle fsb$ 68 d. 20 m.

So is sd 100 parts,

To db 164 parts.

Fourthly, In the Obliquangular Triangle fsa .

There is given the two sides sa 401 parts, and
 sd 100 parts, and the contained Angle $\angle fsa$,

To

To find the other two Angles $fd a$ and $fa d$,
and the third side da .

Then the greater side is fa 401 parts,
And the less side is — fd 100

Then as the Sum of the sides 501

Is to the Difference of the sides 301

So is the Tangent of 55 deg. 50 min.

To the Tangent of 41 31

The Angle $fd a$ — 97 21

The Angle $fa d$ — 14 19

Then as the Sine of the Angle $fa d$ 14 d. 19 m.

Is to the Sine of the Angle afd 68 20

So is fd 100 parts,

To da 376 parts.

Fifthly, In the Obliquangular Triangle BAT.

There is given the Base BA 415 parts, and the
two sides AT 376 parts, and BT 164 parts,
To find the Segments of the Base OA and OB,
and the perpendicular OT.

A 3

The

The greater side is A T 376 parts;

The less side is B T 164 parts;

The Sum of the sides — 540 parts

The Difference of the sides 212

Therefore as the Base BA ——— 415 parts,

Is to the Sum of the sides ——— 540

So is the difference of the sides ——— 212

To the difference of the Seg. of the Base 276

Therefore to BA ——— 415 parts

Add ——— 276

The Sum is 691

The half Sum is A O ——— 345½

And from B A 415 parts,

Subtract ——— 276

The Difference is 139

The half is B O ——— 69½

and

A

The

There

Is

So is

Te

The side BT is 164 parts,
And BO is — 69 $\frac{1}{2}$

The Sum is 233 $\frac{1}{2}$ parts, Log. 2.367356
The Difference is 95 parts, Log. 1.977724

Sum Log. 4.345080

Half Sum Log. 2.172540

Log. of OT 149 parts.

Sixthly, In the Obliquangular Triangle BPA,

There is given the Base BA 415 parts, and
the two sides PA 366 parts and PB 55 parts,

To find Bx and Ax the Segments of the Base
and the perpendicular Px.

The greater side PA 366 parts,

The less side PB — 55

Their Sum is — 421

Their Difference is 311

Therefore as the Base BA 415 parts,

Is to the Sum of the sides 421 parts:

So is the difference of the sides 311

To — 315

B 4

There

Therefore to BA 415 parts
Add 315

The Sum is 730

The $\frac{1}{2}$ is the greater Segment Ax. 365

And from BA 415 parts
Subtract 315

The Difference is 100

The $\frac{1}{2}$ is the less Segment Bx 50

The left side BP 55

The Segment Bx 50

The Sum is 105 Log. 2.021189

The difference is 5 Log. 0.698970

The Sum of the Log. 2.720159

The $\frac{1}{2}$ Sum is the Log. of P x 23, 2.360079 $\frac{1}{2}$

BO 69 parts

Bx 50

Ox 19

Seventh.

Seventhly, In the Right-angled Triangle P G I.

There is given the Hypotenuse P I 1000 parts, and the side P G 19 parts,

To find the side G I.

The greater side P I 1000 parts

The less side P G 19

Their Sum ——— 1019 parts, Log. 2.075447

Their difference 81 parts, Log. 1.903485

The Sum of the Log. 3.984032

The $\frac{1}{2}$ Sum is the Log. of G I 98 parts, 1.992016

Eightly, In the Right-angled Triangle K G O,

There is given K G 164 parts and O G 23 parts,
To find the Angle G K G, and the side O K.

The Proportion is as G K 164 parts,

Is to G O 23 parts :

So is the Radius,

To the Tangent of the Angle G K O 7 d. 59 m.

And

And as the Sine of 7 deg. 59 min.

Is to GO 23 parts:

So is the Radius,

To OK 166 parts.

Ninthly, In the Obliquangular Triangle OKD,

There is given the Base OK 166 parts, and
OD 149 parts, equal to the Radius of the Se-
micircle LDT and KD 98 parts,

To find the Angle OKD.

OD 149 parts

KD 98 parts

Their Sum 247

Their Difference 51

Then as the Base OK 166 parts

Is to the Sum 247

So is the difference 51

To 76

In A

Therefore

[11]

Therefore to OK 166 parts
Add 76

The Sum is 242

The $\frac{1}{2}$ Sum is O γ 121

And from O K 166 parts
Subtract 76

The difference is 90

The $\frac{1}{2}$ difference is K γ 45

Then as DK 98

Is to K γ 45

So is the Sine of 90 degrees,

To the Sine of the Angle K D γ 27 d. 20 m;

The Complement is the Angle DK γ 62 40

Subtract the Angle GK O 7 59

Remains the Angle DK α 54 41.

Tenthly,

Tenthly, In the Right-angled Triangle DKz ,

There is given the Hypotenuse DK 98 parts
and the Angle DKz 54 degrees 41 minutes,

To find the perpendicular Dz and the Base
 Kz :

For as the Sine of DzK 90 deg.
Is to the Hypotenuse DK 98 parts
So is the Sine of the Angle DKz 54 d. 41 m.
To the perpendicular Dz 57 parts.

Eleventhly, In the Right-angled Triangle
 PGQ ,

There is given the Base PG 19 parts,
And the perpendicular GQ 80 parts,

To find the Hypotenuse PQ .

The proportion is as GQ 80 parts,
Is to PG 19 parts:

So is the Radius,

To the Tangent of the Angle PGQ 13 deg.
20 min.

And

And as the Sine of the Angle PQG 13 d. 20 m.

Is to PG 19 parts:

So is the Sine of 90 degrees,

To PQ 82 parts.

Then from NP 164

Take — Nr 57

Remains — rP 107

Twelfthly, To find PC ;

There is given Nr 57, and rM 82, and NP 164. The Proportion is,

As Nr 57

Is to rM 82

So is NP 164

To PC — 236

Lastly,

Lastly, To find **PE**,

The Proportion is, As **PC 236**

Is to **PN 164**

So is **PN 164**

To **PE 114**

Now if the Plane of the Dial be on an upright Wall, the Meridian Line may be found as in the 13th Chapter of the First Part, Figure 14th, and the Hours may be set on the Equinoctial Line by the First Chapter of the Second Part, Fig. 21.

And Six Hours being given, the rest may be found by the Third Chapter of the Second Part.

Or Four Hours and the Equinoctial Line being given, the rest may be

be found by the Ninth Chapter of
the Second Part.

And then the Dial may be drawn
as in the 35th Figure.

F I N I S.
